Evaluation of the Equivalent Ductility of Buildings with Dynamic Soil-Structure Interaction Effects Using Capacity Curves

Evaluación de la ductilidad equivalente de edificios con efectos de interacción dinámica suelo-estructura utilizando curvas de capacidad

Luciano Roberto Fernández Sola
Materials Department,
Metropolitan Autonomus University
lrfs@azc.uam.mx

Dossier

Sumission date: August 6, 2018
Approval date: September 12, 2018

DOI: 10.22201/fa.2007252Xp.2018.18.67943

Abstract

The analysis of buildings under seismic motion with a flexible base must consider two principal aspects of structural displacement: the first, being structural deformation, the second, rigid body behavior. This effect produces a modification of the inelastic behavior of structures. In addition, a consideration of the flexible base may change the distribution of internal forces along the structure that could generate variations in the ductility demands on different structural elements. This article summarizes the results of previous studies on variations in the inelastic behavior of steel and reinforced concrete structures, taking dynamic soil structure interaction into consideration. The response of buildings with a flexible base is compared and contrasted with those with rigid bases. The inelastic behavior of buildings is set out in terms of ductility capacity and demands. Pushover analysis is used to establish inelastic capacity parameters by comparing the capacity curves of buildings with rigid (fixed) and flexible bases. Some comments and general guidelines are made about how base flexibility influences the inelastic behavior of structures.

Keywords: equivalent ductility, capacity curves, inelastic behavior, dynamic soil-structure interaction

Acknowledgements

All results presented in this paper are the product of dissertation research by the following graduate students: David Dávalos Chávez, Juan Enrique Huerta-Écatl and Jorge Hernández Torres. Some results where computed in collaboration with Dr. Edgar Tapia-Hernández. I would like to thank UAM-Azcapotzalco for its funding through Project MA029-13 and CONACyT through the grants given to the aforementioned graduate students.

Resumen
El análisis de las estructuras bajo acciones sísmicas con base flexible debe considerar dos componentes principales del movimiento: uno asociado con la deformación propia de la estructura y el otro relacionado con el movimiento de cuerpo rígido. Adicionalmente, la consideración de la base flexible puede modificar la distribución y magnitud de las fuerzas internas dentro de la estructura y puede, incluso, modificar las demandas de ductilidad en distintos elementos estructurales. En este trabajo se resumen los resultados de estudios previos respecto a la variación del comportamiento inelástico de marcos de acero y de concreto considerando los efectos de interacción dinámica sueloestructura. Se compara la respuesta de las estructuras con base flexible y con base rígida. El comportamiento inelástico se define con base en curvas de capacidad calculadas con análisis estáticos no lineales. Se calculan las demandas de ductilidad y la capacidad de deformación inelástica de las estructuras por medio de las curvas de capacidad. Además, se incluyen algunos comentarios y conclusiones generales acerca de la influencia de la base flexible en el comportamiento inelástico de las estructuras.

Palabras clave: *ductilidad equivalente, curvas de capacidad, comportamiento inelástico, interacción dinámica suelo estructura*

Introduction

The inelastic behavior of structures has become a fundamental aspect of seismic design. It is essential to be able to predict the inelastic deformations of structural elements under seismic motion. Since demands can only be computed by a nonlinear time history analysis of the fully designed structure in conjunction with a specific seismic motion, alternative methods must be used for design procedures. The response spectrum method is by far the most commonly used approach to computing force and deformation demands. For inelastic analysis, the use of the inelastic response spectrum of uniform ductility demand allows us to establish the yield strength required by the structure to control global ductility demand. With this approach, it is possible to choose the maximum ductility demand desired on the structure and then establish the required yield strength. Once the yield strength and maximum ductility demand have been defined, the designer must ensure that the structure will be capable of resisting these demands. The structure's strength and ductility capacity should be greater than demand. Building codes around the world use the well-known approach of yield strength reduction factors (R_μ).

First, the designer chooses the maximum ductility demand desired on the structure (μ) and then the associated R_μ is computed. The relationship between μ and R_μ depends on the structure's dynamic properties and the characteristics of the input motion.[1] With R_μ values, the acceleration demands computed from the elastic response spectrum are reduced, so the required structural strength is defined.

Most of the ideas and hypotheses for this method were developed for systems whose supports are fixed. In addition, response spectrums are built with the maximum responses of Single Degree of Freedom systems (SDOF). The ratio between R" and μ is computed with SDOF systems as well. Under these conditions, all structural responses are represented by a single displacement. The entire displacement of the system is associated with structural deformation, so ductility is defined as the ratio between maximum and yielding displacements.

However, in some cases, the stiffness of the soil foundation system is not enough to constitute a fixed base, so a relative displacement is produced between the foundation and the surrounding soil. Soil structure system displacement includes two principal elements, one the result of structural deformation (u) and other of rigid body behavior (u_0 and θ) as shown in figure 1. The interaction between the soil and the foundation can modify the structure's dynamic properties, excitation characteristics and soil behavior. Those modifications which arise from the joint performance of the soil and the foundation are defined as Dynamic Soil-Structure Interaction (DSSI).

In general, DSSI is calculated through the modification of the structural period (lengthening) and the damping produced by system flexibilization.[2] The structure will therefore be subjected to a modified spectral acceleration demand. Several building codes[3] use the base shear variation associated with the spectral acceleration shift to compute changes in the remaining response quantities (e.g. displacements, element forces, etc). Nevertheless, the presence of rigid body displacement elements modifies the relationship between μ and R_μ.

Since the response spectrum method is based on the response of SDOF systems, the approach for flexible base structures using the response spectrum must represent the soil structure system with an

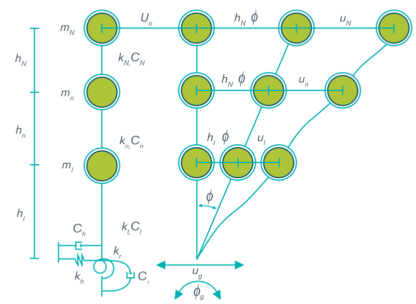

Figure 1: Displacement elements of a structure with a flexible base.

1 See: Anil Kumar Chopra, *Dynamics of Structures: Theory and Applications to Earthquake Engineering* (New Jersey: Prentice Hall, 2001).

2 John P. Wolf, *Dynamic Soil-Structure Interaction* (Prentice Hall, 1985).

3 See: ASCE 7, "Minimum Design Loads for Buildings and Other Structures," *ASCE Standard ASCE/SEI 7-10*, American Society of Civil Engineers, 2010; NBCC, "National Building Code of Canada," *National Research Council of Canada*, Ottawa, 2015; NZS 3101-1, "New Zealand Standard Code of Practice for General Structural Design and Design Loadings for Buildings," *Standards Association of New Zealand*, Wellington, 2006; and MCBC, "Reglamento de construcciones para el Distrito Federal," *Gaceta Oficial del Departamento del Distrito Federal*, Mexico, 2004.

Equivalent Single Degree of Freedom system (ESDOF). Previous studies had characterized the modifications introduced by the flexible base using the concept of an equivalent system with a single degree of freedom (ESDOF).[4] They established the equivalent properties of a single degree of freedom system (fundamental period, damping ratio and ductility) that may reproduce the inelastic response of a multi-degree system with a flexible base. This approach is the one used in several building codes that account for the DSSI.[5] Equivalent ductility ($\tilde{\mu}$) is defined as the ratio of the maximum ($\tilde{u}_u$) and yield displacement ($\tilde{u}_y$) of the ESDOF (equation 1).

$$\tilde{\mu} = \frac{\tilde{u}_u}{\tilde{u}_y}$$

Equation 1

This ductility can be computed as a function of the fundamental period and the ductility of the system with a fixed base (T and μ) and the equivalent period ($\tilde{T}$) with equation 2. This assumes that the ESDOF behaves as a perfectly elastoplastic system without considering post-yielding stiffness. In redundant systems, where many elements contribute to lateral stiffness, capacity curves show a progressive yield, which must be modeled as a bilinear system with post-yield stiffness. The effect of displacement elements due to rigid body behavior on the inelastic branch will be smaller, but not null, as for an elastoplastic model.[6]

Equivalent ductility always yields smaller values than with a fixed base. This does not mean that a structure with a flexible base has a reduced inelastic capacity, as commonly misunderstood. The ductility factor must be corrected due to the modification of the relationship between the yield strength reduction factor (R_μ) and the ductility factor (μ) produced by base flexibility. If elastic forces are reduced by the fixed base yield strength reduction factor without any correction, ductility demands on a structure with a flexible base may be increased.[7]

$$\tilde{\mu} = \left(\frac{T}{\tilde{T}}\right)^2 (\mu - 1) + 1$$

Equation 2

4 See: Emilio Rosenblueth and Daniel Reséndiz, "Disposiciones Reglamentarias de 1987 para tener en cuenta interacción dinámica suelo-estructura," *Series del Instituto de Ingeniería* 509 (1988) and Muberra Eser and others, "Effects of soil-structure Interaction on strength reduction factors," *Procedia Engineering* 14 (2011): 1696-1704.

5 ASCE 7, "Minimum Design Loads for Buildings and Other Structures;" NBCC, "National Building Code of Canada;" NZS 3101-1, "New Zealand standard code of practice for general structural design and design loadings for buildings;" and MCBC, "Reglamento de construcciones para el Distrito Federal."

6 Javier Avilés and Luis Eduardo Pérez Rocha, "Use of Global Ductility for Design of Structure-Foundation systems," *Soil Dynamics and Earthquake Engineering* 31 (2011): 1018-1026, and Luciano Roberto Fernández Sola and Juan E. Huerta Écatl, "Inelastic Behavior of RC Building Considering Dynamic Soil-Structure Interaction," (16 World Conference on Earthquake Engineering, Chile, January 9-13, 2017).

7 Fernández Sola and Huerta Écatl, "Inelastic Behavior of RC Building Considering Dynamic Soil-Structure Interaction."

The use of an ESDOF is very useful and yields good results in many cases. Since just one degree of freedom is used, this procedure implicitly considers the modifications introduced by base flexibility in all structural responses throughout the structure to be linearly equivalent. However, studies have shown that, in some cases, the representation of a flexible base system with multiple degrees of freedom with an ESDOF may not yield good results.[8]

Equivalent ductility can be computed from the capacity curve of structures with a flexible base.[9] This article summarizes the results of prior studies conducted by the author and others on the inelastic behavior of buildings with fixed and flexible bases in order to show the main changes in equivalent ductility and the influence of post-yield stiffness. The capacity curves of buildings with fixed and flexible bases are presented. These curves are computed in two different ways. First, the total displacement of the soil structure system is considered ($\tilde{u}$), including rigid body elements (u_0 and Θ). This set of results is used to compute equivalent ductility. Capacity curves are then computed using only the displacement associated with structural deformation (u) to determine if the structure's inelastic parameters are modified by base flexibility due to P-Δ effects. Results for braced steel frame buildings and reinforced concrete frame buildings are presented.

The influence of post yield stiffness on equivalent ductility

The relationship between R_μ and μ is determined by the percentage of the total displacement produced by rigid body behavior for yield and maximum displacement.[10] Since inelastic displacements are only

8 Armando Barcena and Luis Esteva, "Influence of Dynamic Soil–Structure Interaction on the Nonlinear Response and Seismic Reliability of Multistory Systems," *Earthquake Engineering and Structural Dynamics* 36, no.3 (2007): 327-346; Behnoud Ganjavi and Hong Hao, "Elastic and Inelastic Response of Single- and Multi-Degree-of-Freedom Systems Considering Soil Structure Interaction Effects," (Australian Earthquake Engineering Society 2011 Conference, Barossa Valley, South Australia, 2011); Muberra Eser Aydemir and Cigdem Aydemir, "Overstrength Factors for SDOF and MDOF Systems with Soil Structure Interaction, "*Earthquakes and Structures* 10, no. 6 (2016): 1273-1289; Mehdi Ghandil and Farhad Behnamfar, "Ductility Demands of MRF Structures on Soft Soils Considering Soil-Structure interaction," *Soil Dynamics and Earthquake Engineering* 92, (2017): 203-214.

9 Fernández Sola and Huerta Écatl, "Inelastic Behavior of RC Building Considering Dynamic Soil-Structure Interaction," and Luciano Roberto Fernández Sola and others, "Respuesta inelástica de marcas de acero interacción inercial suelo-estructura", *Ingeniería sísmica 92*, (2015): 1-21.

10 See: Avilés and Pérez Rocha, "Use of Global Ductility for Design of Structure-Foundation Systems."

related to u, the ratio between displacement produced by rigid body elements (u_0 and Θ) and u is different for yield and maximum displacement. In order to take this effect into consideration, Fernández Sola and Huerta Écatl have proposed the following procedure:[11] using equation 1, the maximum displacement of ESDOF system ductility ($\tilde{u}_u$) is expressed as equation 3.

Where:

$$\tilde{u}_u = \tilde{\mu}\tilde{u}_y$$

Equation 3.

- $\tilde{u}_u = u_{rb}^u + u^u$ - Maximum displacement of the ESDOF
- u_{rb}^u - Components of displacement due to rigid body at maximum displacement
- u^u - Structural deformation at maximum displacement
- $\tilde{u}_y = u_{rb}^y + u^y$ - Yield displacement of the ESDOF
- u_{rb}^y - Components of displacement due to rigid body at yield displacement
- u^y - Structural deformation at yield displacement

Expressing $\tilde{\mu}$ in terms of rigid body displacement and structural deformation yields (equation 4)

Equation 4 can be expressed in terms of the deformation of the structure (u^u and u^y)as equation 5:

To compute the ductility of the structure (μ), only the displacement produced by structural deformation should be considered. Consequently, the relationship between ESDOF system ductility with a flexible base ($\tilde{\mu}$) and ductility in the structure (μ) can be defined as equation 6:

$$u_{rb}^u + u^u = \mu\left(u_{rb}^y + u^y\right)$$

Equation 4

$$u^u\left(1+\frac{u_{rb}^u}{u^u}\right) = \tilde{\mu}u^y\left(1+\frac{u_{rb}^y}{u^y}\right)$$

Equation 5

$$\mu = \frac{u^u}{u^y} = \tilde{\mu}\frac{\left(1+\frac{u_{rb}^y}{u^y}\right)}{\left(1+\frac{u_{rb}^u}{u^u}\right)} \rightarrow \tilde{\mu} = \mu\frac{\left(1+\frac{u_{rb}^u}{u^u}\right)}{\left(1+\frac{u_{rb}^y}{u^y}\right)}$$

Equation 6

11 See: Fernández Sola and Huerta Écatl, "Inelastic Behavior of RC Building Considering Dynamic Soil-Structure Interaction."

For elastoplastic systems, rigid body displacements $u^{u}{}_{rb}$ and $u^{y}{}_{rb}$ are equal. Substituting $u_{rb}{}^{u} = u_{rb}{}^{y} = u_{rb}$ and using $u_{rb} = \tilde{u}_{y} - u^{y}$, after some algebra, proves that equation 6 yields equation 2. However, a system's post-yield stiffness leads to larger values of $\tilde{\mu}$, as shown by Avilés and Pérez Rocha.[12]

Equivalent ductility from capacity curves

Equivalent ductility ($\tilde{\mu}$) can be computed from the capacity curves of the soil-structure system using total displacement as the displacement of the ESDOF system ($\tilde{u}$). Results for three different types of structures are presented.

First, the results reported by Fernández Sola, Tapia Hernández and Dávalos Chávez[13] correspond to the inelastic behavior of individual braced steel frames with one and two braced bays and unbraced frames. All frames are part of the same building. Three different buildings of 8, 12 and 16 storeys are analyzed. All buildings are built on a soft clay layer with a shear wave velocity of Vs=65 m/s. Two types of foundations are used for each building, one that consists of a mat foundation and the other of frictional piles.[14]

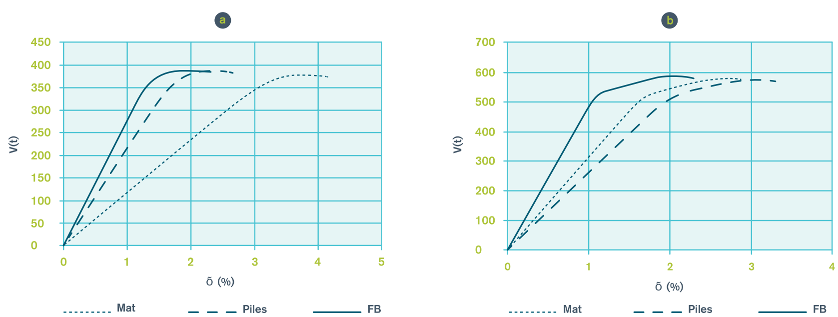

Figure 2: capacity curves with total displacement (u) for unbraced frames on a fixed base (FB), mat foundation (Mat) and pile foundation (Piles) for a) 8-storey and b) 16-storey buildings.

12 Áviles and Pérez Rocha, "Use of Global Ductility for Design of Structure Foundation System."

13 Fernández Sola and others, "Respuesta inelástica de marcas de acero con interacción inercial suelo-estructura."

14 See: Fernández Sola and others, "Respuesta inelástica de marcos de acero con interacción inercial suelo-estructura."

Capacity curves built with total displacement ($\tilde{u}$) are shown in figure 2 for the unbraced frames of the 8 and 16 storey buildings. As expected, frames with DSSI effects are more flexible. This means that $\tilde{u}_y$ and $\tilde{u}_u$ are larger than for the FB. For the 8 storey building, the pile foundation is stiffer than the mat foundation. The opposite happens for the 16 storey building. Yield and maximum base share are very similar for the frames with FB and DSSI, so for these cases, base flexibility does not influence overall resistance.

Values for $\tilde{\mu}$ are shown in table 1. These values are computed using bilinear primary capacity curves ($\tilde{\mu}_{cc}$). Yield displacement is defined at the intersection of elastic and inelastic branches. Fernández Sola,[15] report that the overstrength factors for these frames are 1.07 and 1.10 for the 8 and 16 storey frames respectively. This means that both frames behave almost as perfect elastoplastic systems.

Height	$\tilde{\mu}_{CC}$			$\tilde{\mu}_{eq2}$	
	FB	Piles	Mat	Piles	Mat
8-storey	1.66	1.51	1.29	1.49	1.27
16-storey	1.89	1.51	1.60	1.49	1.59

Table 1- Equivalent ductility $\tilde{\mu}$ computed from capacity curves ($\tilde{\mu}_{55}$) and with equation 2 ($\tilde{\mu}_{67.-}$)

From the results seen in table 1, it can be seen that, as the base becomes more flexible (mat foundation for the 8 storey frame and pile foundation for the 16 storey frame), $\tilde{\mu}$ values get smaller. Since the frames presented behave almost as elastoplastic systems with very low post-yield stiffness, the $\tilde{\mu}$ values computed from capacity curves are very similar to the values computed using equation 2 ($\tilde{\mu}_{eq2}$).

15 Fernández Sola and others, "Respuesta inelástica de marcas de acero interacción inercial suelo-estructura."

In order to compare the ductility of the structure (μ), capacity curves are computed using the displacement associated with the deformation of the structure (u). These curves are shown in figure 3. Capacity curves are very similar for the three booth frames. This indicates that the inelastic behavior of the structural system is basically unchanged by base flexibility, that is, if the strength reduction factor is defined by $\tilde{\mu}$, the ductility of the structural system is the same for all cases. This is the reason for using $\tilde{\mu}$ values to compute R_{μ}. There are some very small differences in frame resistance. Frames with a flexible base experience a reduction of maximum base shear values. In addition, a dropdown in terms of the elastic stiffness of frames with DSSI is observed. This can be associated with the amplification of P-Δ effects produced by base flexibility.

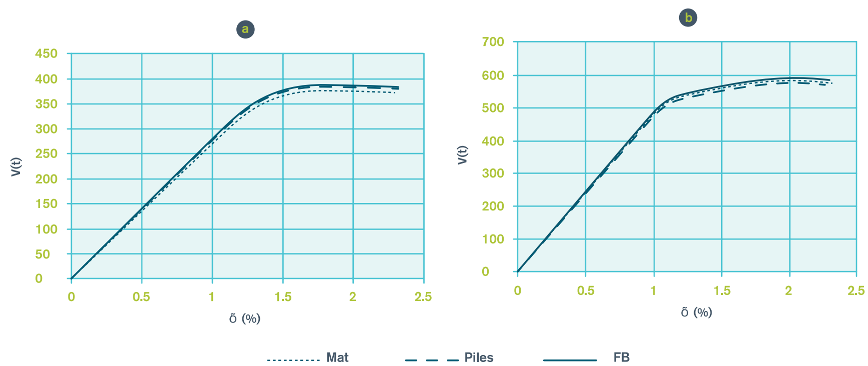

Figure 3: capacity curves with structural deformation (u) for unbraced frames on a fixed base (FB), mat foundation (Mat) and pile foundation (Piles) of a) 8 storey and b) 16 storey buildings.

In order to study the potential strength and stiffness reduction produced by the amplification of P-Δ effects, a parametric analysis was performed. The capacity curves of the 16-storey frame with a pile foundation were computed using extremely reduced values of soil foundation system stiffness. These values are unreal and are used to amplify P-Δ effects. Figure 4 shows the capacity curves computed with u using 100%, 20% and 5% soil pile foundation stiffness. Similar results for the other cases can be found in Fernadez Sola, Tapia Hernández and Dávalos Chávez[16] Drastic base stiffness reduction produces a reduction in elastic stiffness and yield base shear. It is worth remembering that these capacity curves show only the deformation of the structure without rigid body

16 Fernandez Sola and others, "Respuesta inelástica de marcas de acero con interacción inercial suelo-estructura."

elements. These effects are produced by the increase of P-Δ effects due to the amplification of the relative displacement between the ends of the columns produced by base rotation. These effects have been examined in single steel columns in previous studies.[17]

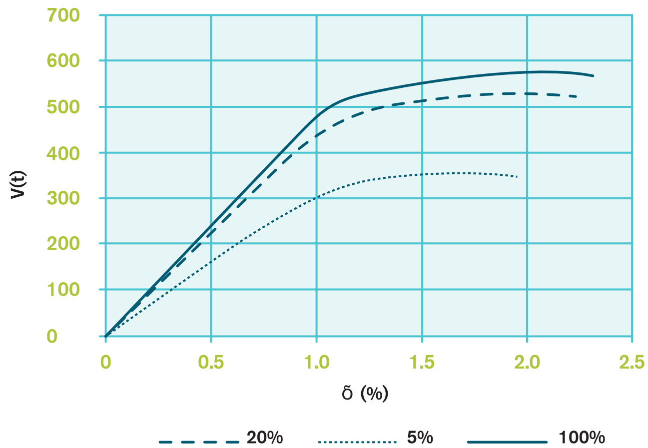

Figure 4: capacity curves with structural deformation (u) for unbraced frames with 100%, 20% and 5% soil foundation stiffness.

In order to show the influence of overstrength on the inelastic behavior of systems with a flexible base, the results presented by Huerta Écatl and Fernández Sola[18] are shown. In this study, a 10 storey building with reinforced concrete (RC) frames and a mat foundation is used. Three different soil stiffness values are considered (Vs=70, 100 and 250 m/s). Again, capacity curves with total displacement ($\tilde{u}$) and with structural deformation (u) are computed (figure 5). In this study, results are obtained for the whole building and not for individual frames.

For the steel frames presented previously, as the soil becomes more flexible, $\tilde{u}_u$ and $\tilde{u}_y$ values become higher. The variation in the relationship between $\tilde{u}_u$ and $\tilde{u}_y$ produces changes to $\tilde{\mu}$. When the deformation of the structure is analyzed, capacity curves for all soil conditions

17 Sergio Ermenegildo Jacinto and Luciano Roberto Fernández Sola, "Influencia de los efectos P-Delta al considerar base flexible (ISE) en columnas deacero," (XX Congreso Nacional de Ingeniería Sísmica, Mexico, 2015).

18 Juan E. Huerta Écatl, "Evaluación de la interacción dinámica suelo-estructura en el Comportamiento Inelástico de un edificio de concreto reforzado," Master's Dissertation, Postgraduate in Structural Engineering, UAM Azcapotzalco, 2015; and Luciano Roberto Fernández Sola and Juan E. Huerta Écatl, "Inelastic Behavior of RC Building Considering Dynamic Soil-Structure Interaction."

Vs(m/s)	$\tilde{\mu}_{55}$	$\tilde{\mu}_{67.-}$	$\tilde{\mu}_{67.9}$
∞(FB)	2.67	2.67	2.67
250	2.54	2.50	2.54
100	2.12	2.00	2.15
70	1.85	1.66	1.89

Table 2- Equivalent ductility ($\tilde{\mu}$) computed using capacity curves ($\tilde{\mu}_{55}$), equation 2 ($\tilde{\mu}_{67.-}$) and equation 6 ($\tilde{\mu}_{67.9}$).

are nearly identical. In this case, differences among capacity curves for fixed and flexible bases computed with structural deformation (u) are smaller than those for steel frames. This is expected, since P-Δ values are expected to be less important on RC frames. On the other hand, these systems develop greater overstrength factors than steel frames, around 1.25.[19] As mentioned above, overstrength influences the values of $\tilde{\mu}$.

$\tilde{\mu}$ values computed using capacity curves ($\tilde{\mu}_{55}$), equation 2 ($\tilde{\mu}_{67.-}$) and equation 6 ($\tilde{\mu}_{67.9}$) are shown in table 2.[20]

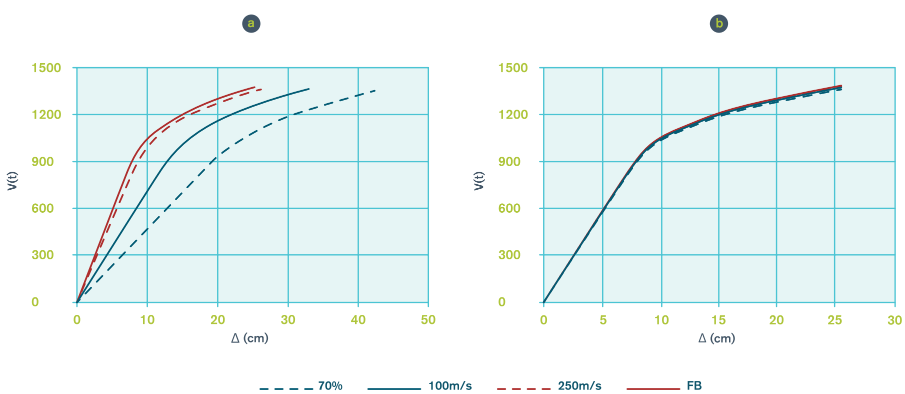

Figure 5: capacity curves for RC buildings on different soil types (Vs=70, 100 and 250 m/s) and a fixed base with a) structural deformation (u) and b) total displacement ($\tilde{u}$) (figures 5).

19 Details on the role of each displacement element can be found in Huerta Écatl, "Evaluación de la interacción dinámica suelo-estructura en el comportamiento inelástico de un edificio de concreto reforzado."

20 See: Huerta Écatl, "Evaluación de la interacción dinámica suelo-estructura en el comportamiento inelástico de un edificio de concreto reforzado."

As for the steel frames, table 2 shows that, as the soil becomes more flexible, $\tilde{\mu}$ values are lower. In addition, it can be seen that $\tilde{\mu}$ values computed using an elastoplastic model ($\tilde{\mu}_{eq.2}$) yield lower values than those computed directly from capacity curves ($\tilde{\mu}_{CC}$) due to the post-yield stiffness of the structure. When $\tilde{\mu}$ is computed taking into consideration the overstrength of the structure ($\tilde{\mu}_{eq.6}$), values are similar to those computed directly from the capacity curves. From these results, it is clear that elastoplastic models can overestimate changes in ductility due to base flexibility.[21]

Hernández Torres[22] studied the inelastic behavior of steel buildings with braced frames at different heights (4, 7 and 10-storey) with both fixed and flexible bases. Mat foundations on a soft soil of V_s=80 m/s were used. Capacity curves considering total displacement ($\tilde{u}$) are shown in figure 6. This analysis is performed for the whole building and not for individual frames.[23] In these cases, the higher the structure, the larger the DSSI effects. Similar effects to those from the previous results can be observed. These structures develop large overstrength factors. μ values computed from capacity curves ($\tilde{\mu}_{67.9}$), equation 2 ($\tilde{\mu}_{55}$) and equation 6 ($\tilde{\mu}_{67.-}$) are shown in table 3.

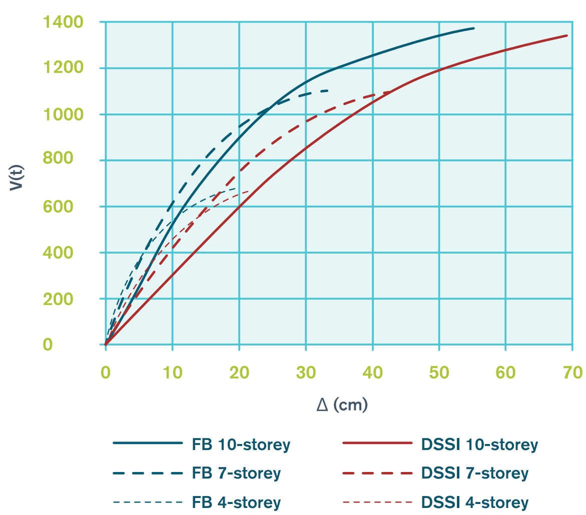

Figure 6: Capacity curves for steel buildings with braced frames with different heights (4, 7 and 10 storeys).

21 As previously shown in Áviles and Pérez Rocha, "Use of Global Ductility for Design of Structure Foundation Systems."

22 See: J. Hernández Torres, "Comportamiento estructural de eficios de acero con base flexible," Master's Dissertation, Postgraduate in Structural Engineering, UAM-Azcapotzalco, 2017.

23 Fernández Sola and others, "Respuesta inelástica de marcas de acero con interacción inercial suelo-estructura".

As for all previous cases, equivalent ductility ($\tilde{\mu}$) is lower than structural ductility ($\tilde{\mu}$). Since these buildings exhibit significant post-yield stiffness, equivalent ductility computed with the elastoplastic model ($\tilde{\mu}_{eq.2}$) yields lower values than those computed directly from capacity curves ($\tilde{\mu}_{CC}$). Equation 6 predicts equivalent ductility better since it explicitly takes post-yield stiffness into account.

Height	μ	$\tilde{\mu}_{55}$	$\tilde{\mu}_{e67.-}$	$\tilde{\mu}_{67.9}$
4-storey	3.43	2.81	2.59	2.84
7-storey	2.81	2.25	2.07	2.27
10-storey	2.75	2.19	2.01	2.19

Table 3- Equivalent ductility ($\tilde{\mu}$) computed from capacity curves ($\tilde{\mu}_{55}$), Equation 2 ($\tilde{\mu}_{67.-}$), and Equation 6 ($\tilde{\mu}_{67.9}$).

Final remarks

This article has presented a summary of studies of the inelastic behavior of flexible base systems and explored the use of equivalent ductility ($\tilde{\mu}$) for the inelastic design of structures with dynamic soil structure interaction effects (DSSI) with the response spectrum method. The relationship between the yield strength reduction factor ($R\mu$) and structural ductility (μ) is modified by DSSI. Since the response spectrum method is based on the response of single degree of freedom systems, the use of $\tilde{\mu}$ is necessary in order to keep μ demands within design values.

Post-yield stiffness plays an important role in $\tilde{\mu}$ variations. Procedures included in building codes are based on a perfect elastoplastic equivalent system. $\tilde{\mu}$ values computed with this procedure tend to be lower than those computed for systems with post-yield stiffness.

Equivalent ductility can be computed directly from the systems' capacity curves. Results for individual steel frames, reinforced concrete (RC) buildings and steel buildings with braced frames are shown. Capacity curves are computed with two sets of results: one using the complete system displacement, including rigid body displacements, and one using only the deformation of the structure. μ values are computed with the first set of results and μ values are computed with the second set of results.

It has been confirmed that the $\tilde{\mu}$ value is smaller than the μ value for all cases with DSSI effects. Elastic stiffness is always reduced by base flexibility. Yield and maximum base shears and overstrength factors are very similar. On the other hand, capacity curves that only take structural deformation into consideration are almost entirely unmodified by DSSI effects. Very small reductions in elastic stiffness and strength can be noticed on steel frames, a product of P-Δ effects.

When structures exhibit post-yield stiffness, μ values computed with elastoplastic models are lower than those computed directly from capacity curves. On the other hand, μ values computed with equations that take structural overstrength into consideration are very close to those computed directly from capacity curves.

References

ASCE 7, "Minimum design loads for buildings and other structures," *ASCE Standard ASCE/SEI 7-10, American Society of Civil Engineers*, 2010.

Avilés, Javier and Luis E. Perez Rocha. "Use of Global Ductility for Design of Structure-Foundation Systems," *Soil Dynamics and Earthquake Engineering* 31, (2011): 1018-1026.

Avilés, Javier and Luis E. Perez-Rocha. "Soil-structure Interaction in Yielding Systems," *Earthquake Engineering and Structural Dynamics* 32, no. 11, (2005): 1749-1771.

Aydemir Muberra, Eser and others. "Effects of Soil-Structure Interaction on Strength Reduction Factors," *Procedia Engineering* 14, (2011): 1696-1704.

_____ and Cinden Aydemir. "Overstrength Factors for SDOF and MDOF Systems with Soil-Structure Interaction," *Earthquakes and Structures* 10, no.6, (2016): 1273-1289.

Barcena, Armando and Luis Esteva. "Influence of Dynamic Soil–Structure Interaction on the Nonlinear Response and Seismic Reliability of Multistory Systems," *Earthquake Engineering and Structural Dynamics* 36, no. 3, (2007): 327-346.

Chopra, Anil Kumar. *Dynamics of Structures: Theory and Applications to Earthquake Engineering*. New Jersey: Prentice Hall, 2001.

Ermenegildo Jacinto, Sergio and Luciano Roberto Fernández Sola. "Influencia de los efectos P-Delta al considerar base flexible (ISE) en columnas de acero," XX Congreso Nacional de Ingeniería Sísmica, Mexico, 2015.

Fernández Sola, Luciano Roberto and Juan E. Huerta Écatl. "Inelastic Behavior of RC Building Considering Dynamic Soil-Structure Interaction." 16 World Conference on Earthquake Engineering, Chile, January 9-13, 2017.

Fernández-Sola, Luciano Roberto and others. "Respuesta inelástica de marcos de acero con interacción inercial sueloestructura," *Ingeniería sísmica* 92, (2015): 1-21.

Ganjavi, Behnoud and Hong Hao. "Elastic and Inelastic Response of Single- and Multi-Degree-of-Freedom Systems Considering Soil-Structure Interaction Effects," Australian Earthquake Engineering Society 2011 Conference, Barossa Valley, South Australia, 2011.

Ghandil, Mehdi and Farhad Behnamfar. "Ductility Demands of MRF Structures on Soft Soils Considering Soil-Structure Interaction," *Soil Dynamics and Earthquake Engineering* 92, (2017): 203-214.

Hernández Torres, J. "Comportamiento estructural de edificios de acero con base flexible," *Master's Dissertation, Postgraduate in Structural Engineering*, uam Azcapotzalco, 2017.

Huerta Écatl, Juan E. "Evaluación de la interacción dinámica suelo-estructura en el comportamiento inelástico de un edificio de concreto reforzado," *Master's Dissertation, Postgraduate in Structural Engineering*, uam Azcapotzalco, 2015.

mcbc, "Reglamento de construcciones para el Distrito Federal," *Gaceta Oficial del Departamento del Distrito Federal*, Mexico, Mexico, 2004.

nbcc. "National Building Code of Canada," *National Research Council of Canada*, Ottawa, 2015.

nzs 3101-1. "New Zealand Standard Code of Practice for General Structural Design and Design Loadings for Buildings," *Standards Association of New Zealand*, Wellington. 2006.

Rosenblueth, Emilio and Daniel Reséndiz. "Disposiciones reglamentarias de 1987 para tener en cuenta interacción dinámica sueloestructura," *Series del Instituto de Ingeniería* 509, (1988).

Wolf, John P. *Dynamic soil-structure interaction*. New Jersey: Prentice Hall, 1985.

Luciano Roberto Fernández Sola
lrfs@azc.uam.mx

A civil engineer with a Bachelor's degree from the National Polytechnic Institute (ipn) Higher School of Engineering and Architecture (2005) and a Master's and Doctorate in Engineering (2007 and 2011, respectively) with a specialization in structural engineering from the National Autonomous University of Mexico (unam), graduating with an honorable mention on both of the latter occasions.

A specialist in the seismic behavior of structures and foundations and in dynamic soil-structure interactions. He has been an author and coauthor of a variety of publications, including popularization articles, papers presented at national and international conferences, articles in indexed journals and research reports. He has also participated in interinstitutional projects and interuniversity networks and served as a consultant for the private sector.

A research professor at the Structural Engineering Department of the Metropolitan Autonomous University's Azcapotzalco campus, where he has taught undergraduate and graduate classes since 2011. He has been a member of the National Research System at the candidate level from 2013 to 2017 and a professor with a "sought-after" rating in the Teaching Development Program (prodep) since 2013.

He served as the coordinator for the Mexican College of Civil Engineers' operations center during the evaluation of the damage caused by the September 19, 2007 earthquake in Mexico City.

He is currently a member of the Mexican Geotechnical Engineering Society's Soil-Structure Interaction Committee, secretary of the Board of Directors of the Mexican Structural Engineering Society, member of the Competitive Intelligence Committee of the Alliance for the Promotion of Infrastructure Research for the Development of Mexico (Alianza FiiDEM), president of the Continuing Education Committee of the Mexican Structural Engineering Society and coordinator of the Bachelor's in Civil Engineering program at the UAM-Azcapotzalco.

He has also served on the working group that is giving comments on the Complementary Technical Regulations for Seismic Design for Mexico City's 2017 construction regulations.

Ingeniero Civil por la Escuela Superior de Ingeniería y Arquitectura del Instituto Politécnico Nacional en 2005. Maestro en Ingeniería en 2007 y Doctor en Ingeniería en 2011 con especialidad en estructuras por la DEPFI de la Universidad Nacional Autónoma de México graduado con mención honorífica en ambas ocasiones.

Especialista en el comportamiento sísmico de estructuras y cimentaciones y en interacción dinámica suelo estructura. Ha participado como autor y coautor en diversas publicaciones incluyendo artículos de divulgación, de congresos nacionales e internacionales, en revistas indexadas y reportes de investigación de proyectos. Adicionalmente ha participado en proyectos interinstitucionales y en redes interuniversitarias. De manera adicional, ha desarrollado diversos proyectos de consultoría para la industria.

Profesor investigador en la Universidad Autónoma Metropolitana unidad Azcapotzalco en el área de estructuras impartiendo cursos a nivel de licenciatura y posgrado desde 2011. Miembro del Sistema Nacional de Investigadores con nivel de candidato de 2013 a 2017 y es profesor con perfil deseable del PRODEP desde 2013.

Participó como coordinador del centro de operaciones del Colegio de Ingenieros Civiles de México durante las labores de reconocimiento de daños producidos por el sismo del 19 de septiembre del 2017 en la CDMX.

Actualmente es miembro del comité de Interacción Suelo-Estructura de la Sociedad Mexicana de Ingeniería Geotécnica, secretario de la mesa directiva de la Sociedad Mexicana de Ingeniería Estructural, miembro de la comisión de inteligencia competitiva de la Alianza FiiDEM, presidente del comité de educación continua y estudiantes de la Sociedad Mexicana de Ingeniería Estructural y coordinador de la Licenciatura en Ingeniería Civil de la UAM-Azcapotzalco.

Adicionalmente participa en el grupo de trabajo que está desarrollando los comentarios a la "Norma Técnica Complementaria de Diseño por Sismo" del Reglamento de Construcciones de la Ciudad de México en su versión 2017.