Entreciencias: Diálogos en la Sociedad del Conocimiento

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Año 12, Número 26, Reseña 1: 1-4. Enero - Diciembre 2024
e-ISSN: 2007-8064

http://revistas.unam.mx/index.php/entreciencias

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Una revisión crítica del artículo

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DOI:10.22201/enesl.20078064e.2024.26.87795
e25.87795

Nicolás Guerrero Chávez [a*]
https://orcid.org/0000-0001-6147-5200
Gilberto González Gómez [b*]
https://orcid.org/0000-0001-8550-888X
Karla Anhel Camarillo Gómez [c*▲]
https://orcid.org/0000-0001-6968-9772

Fecha de recepción: 14 de febrero de 2024.
Fecha de aceptación: 8 de marzo de 2024.
Fecha de publicación: 21 de marzo de 2024.

▲Autora de correspondencia
karla.camarillo@itcelaya.edu.mx

*Tecnológico Nacional de México en Celaya

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The article under review explores a mathematical model used with a mixture of volatile organic compounds (VOC) in the biofilm and gaseous phase to investigate various aspects of the biofiltration process. The homotopy perturbation method is used to observe the solution of the model, which is compared to solutions by numerical methods (Li *et al.*, 2023).

The homotopy perturbation method is relatively new for solving non-linear differential equations (Filobello-Nino *et al.*, 2014). It has been evaluated in previous studies, such as the 2015 study by Sivakumar et al., for the solution of a biofilter based on mass transfer. In 2020, Dharmalingam et al. used the method to solve a mathematical model of a bacteria- and fungi-based biofilter. In 2021, Joy Salomi *et al.* studied the solution of non-linear equations in synthesizing n-aminopiperidine.

This document aims to analyze the accuracy of the homotopy perturbation method in solving non-linear differential equations and compare it with the previously mentioned works. Li *et al.* presented a mathematical model of the gas phase and the biofilm for the biofiltration of toluene and n-propanol in their research in 2023. The homotopy perturbation technique is used to solve non-linear partial differential equations. The proposed solution is validated by comparing it with numerical simulations in MATLAB®.

The biofilter used for the research consists of an acrylic tube with an internal diameter of 19.4 cm and a height of 94 cm. Three sampling points are used to collect bed material for analysis. The effective volume of the biofilter is 12.6 l with a total packing height of 42.5 cm. A humidifier maintains an adequate humidity level and an airflow of between 5 and 10 l/min.

The mathematical model of the biofilter performance proposed is based on the mathematical model of Dixit *et al.* in 2011, which includes the microbial kinetics for mixtures of toluene and n-propanol. The proposed model considers the phenomena of convection, absorption, diffusion, and biodegradation. To describe the microbial kinetics, Monod kinetics is used with the inclusion of cross-inhibition effects of the substrate-contaminant mixture.

Four second-order partial differential equations are established: two for the biofilm phase, which describe the diffusion of the compound and the microbial kinetics through the biofilm; one for toluene, and one for n-propanol; and two for the gas part, which describes the concentration of toluene and n-propanol in the gas inside the biofilter. The equations are then converted to dimensionless units.

The homotopy perturbation method is used to solve the model, which reduces the non-linear differential equation to a series of linear differential equations. It is studied under two scenarios: one in a transient state and another in a steady state. In the first one, two scenarios are assumed, one where the microbial kinetics are saturated and another where they are not. For the steady state, it is assumed that there is no net growth or decay in the biofilm for a specific concentration.

A comparison is made in the elimination of toluene and n-propanol in the biofilm phase to observe if there is any difference in the degradation of each contaminant. Finally, the solution of the non-linear partial differential equations by the homotopy perturbation method is compared with numerical simulations performed in MATLAB® to validate the accuracy of the solution.

In conclusion, with the different simulations and parameter variations, it has been demonstrated that, for the biofilm phase, by increasing the value of the parameter that represents the influence of n-propanol on the bio-

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degradation of toluene (α), the concentration of toluene (*Sr*) increases.

In addition, if the parameter value that depends on the inlet concentration of toluene (β) is increased, the concentration of toluene in the system also increases. It has also been observed that the concentration of toluene increases by decreasing the parameter value that depends on the biomass thickness (ϕ). Likewise, the concentration of n-propanol increases if β_1 is increased and ϕ_1 is decreased.

It was observed in the gas phase that increasing α, β, and A while decreasing ϕ and A_1 leads to a rise in the concentration of toluene. Similarly, the concentration of n-propanol rises when increasing β_1 and A, and decreasing ϕ_1 and A_2.

A comparison of the concentration profiles between toluene and n-propanol led to the conclusion that high loads of n-propanol can inhibit toluene degradation; meanwhile, high loads of toluene do not negatively affect the decomposition of n-propanol.

Furthermore, the solutions using the homotopy perturbation method were identical to those obtained using the numerical solution, so using the homotopy perturbation method as a solution to non-linear partial differential equations is valid.

The results of Li *et al.* in 2023 were almost identical to the simulations and experimental data presented by Sivakumar *et al.* in 2015. That is, both present a mathematical biofilter model based on mass transfer and parameter estimation using the homotopy perturbation method, which was then solved by the finite element method. The results presented were almost identical to the simulations and experimental data.

In 2020, Dharmalingam *et al.* developed a model of a biofilter based on bacteria and fungi. Their study aimed to determine the accuracy level of solving non-linear differential equations using the homotopy perturbation method. Also, when the authors compared their results with those obtained by numerical methods, they found almost identical values, demonstrating that the solution method is ideal for non-linear differential equations.

In 2021, Salomi *et al.* studied the solution of non-linear equations using the homotopy perturbation method in the synthesis of n-aminopiperidine. The accuracy level of this work agreed with those works mentioned above. Although the research of Li *et al.* in 2023 proved that the homotopy perturbation method successfully solved non-linear partial differential equations, some areas of opportunity in the presented article can be pointed out. Mainly, the references used to cite relevant and omitted information in the analyzed article have no correspondence or do not exist in the references mentioned. In conclusion, the solution of the mathematical model of biofilter using the homotopy perturbation method presented high accuracy with the numerical solutions performed previously.

Finally, the works compared with the Li *et al.* in 2023 have not the same mathematical model, but the same solution method; all of them reported results very similar to those of reference, demonstrating that the new homotopy perturbation method turns out to be promising for the solution to non-linear equations by its level of accuracy in the results obtained.

Acknowledgments

Support from the National Council of Science and Technology (CONACyT) for postgraduate studies of the first author (identification number 861513).

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Author´s Notes

[a] Master of Science in Mechanical Engineering from the Tecnológico Nacional of Mexico in Celaya, currently pursuing a doctoral program in Engineering Sciences at the Tecnológico Nacional of Mexico in Celaya. His main research areas include bioprocess modeling and sustainable energies. Email: d2003003@itcelaya.edu.mx. Orcid: https://orcid.org/0000-0001-6147-5200

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Guerrero Chávez, N., González Gómez, G., Camarillo Gómez, K.A., (2023). Biofiltros como alternativa para el control de aire contaminado. En *Artículos del Congreso Internacional de Investigación Academia Journals Celaya 2023*, Tomo 6, Ingenierías, (pp. 160-164). Celaya, Gto. Academia Journals. https://www.academiajournals.com/s/Tomo-06-Ingenierias-Articulos-Academia-Journals-Celaya-2023.pdf

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[b] Ph. D. of Science in Chemical Engineering from the Tecnológico Nacional of Mexico in Celaya, currently serving as Academic Subdirector at the Tecnológico Nacional of Mexico in Celaya. His primary research areas include advanced mathematics for engineering and applied electrochemistry. Email: gilberto.gonzalez@itcelaya.edu.mx. Orcid: https://orcid.org/0000-0001-8550-888X

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[c] Ph. D. of Science in Electrical Engineering from the Tecnológico Nacional of Mexico in La Laguna, currently serving as Professor-Researcher in the Department of Mechanical Engineering at the Tecnológico Nacional of Mexico in Celaya. Her main research areas include modeling and control of robots, control of nonlinear systems, analysis of stability of nonlinear systems, development of

DOI: 10.22201/enesl.20078064e.2024.26.87795

rehabilitation systems, technology development for drones, robots, and autonomous vehicles, and automation and vision control applied to industrial processes. Email: karla.camarillo@itcelaya.edu.mx. Corresponding author. Orcid: https://orcid.org/0000-0001-6968-9772

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