Educación Química

Didáctica de la Química

Dimensional Analysis in Chemistry Textbooks 1900-2020 and an Algebraic Alternative

Análisis dimensional en los libros de texto de química de 1900-2020 y una alternativa algebraica

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Resumen

El capítulo sobre unidades y mediciones es prácticamente indispensable en los libros de texto de química moderna, y dentro de él, el método de factor de conversión se ha convertido en sinónimo de análisis dimensional. En este artículo, nuestro objetivo es investigar si esta necesidad ha sido consistente a lo largo de la historia de los libros de texto de química general. También demostraremos que equiparar el concepto de análisis dimensional con la técnica de factor-etiqueta es incorrecto, ya que existe una alternativa algebraica. Para determinar la relación del capítulo sobre unidades y mediciones con el libro de texto, examinamos 140 libros desde la década de 1900 hasta la actualidad, encontrando que este capítulo y sus algoritmos fueron gradualmente introducidos durante la transición de las décadas de 1950 a 1970, acompañados de otros cambios fundamentales en los textos. Además, presentamos una alternativa al método de factor de conversión, llamada sustitución algebraica, que ofrece una mayor diversidad de técnicas para agilizar cálculos específicos. Sin embargo, dominar todas estas técnicas requiere tiempo y habilidad. Esto demuestra que el método de factor de conversión no es la única opción para realizar análisis dimensional. Por lo tanto, no debemos confundir una técnica o algoritmo con la base filosófica subyacente del análisis dimensional.

Palabras clave: análisis dimensional, método de factor-etiqueta, unidades y mediciones, libros de texto, algoritmos.

Abstract

The chapter on units and measurements is practically indispensable in modern chemistry textbooks, and within it, the factor-label method has become synonymous with dimensional analysis. In this paper, we aim to investigate whether this necessity has been consistent throughout the history of general chemistry textbooks. We will also demonstrate that equating the concept of dimensional analysis with the factor-label technique is incorrect, as an algebraic alternative exists. To determine the relationship of the units and measurements chapter with the textbook, we examined 140 books from the 1900s to the present day, finding that this chapter and its algorithms were gradually introduced during the transition from the 1950s to the 1970s, accompanied by other fundamental changes in the texts. Additionally, we present an alternative to the factor-label method, called algebraic substitution, which offers a greater diversity of techniques to expedite specific calculations. However, mastering all these techniques requires time and skill. This shows that the factor-label method is not the only option for performing dimensional analysis. Therefore, we must not confuse a technique or algorithm with the underlying philosophical foundation of dimensional analysis.

Keywords : dimensional analysis, factor-label method, units and measurements, textbooks, algorithms.

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Introduction

The chapter on units and measurements is a common feature in modern chemistry textbooks, often equating dimensional analysis with the factor-label method used to apply it (Ellis, 2013). Dimensional analysis serves three main purposes:

1. **To establish equivalence across different measurement systems** (Speiring, 2001; Whittet & Nixon, 1964; Wrigley, 2011), as neglecting this can lead to catastrophic or costly outcomes (Lloyd & Writer, 1999);
2. **To express a measurement in a multiple or submultiple that aligns with the context of what is being measured**; and
3. **To ensure consistency in equations**, where units must cancel appropriately to produce a coherent result that matches the dependent variable in the formula.

Any relatively modern chemistry textbook, from the past 10 to 20 years, reveals that the chapter on units and measurements is ubiquitous, typically presented in the opening sections of the text. This strategic positioning underscores its fundamental importance, as it is often the first concept introduced in chemistry. This consistent placement highlights how essential it is for students to grasp the foundational understanding of units and dimensional analysis before progressing further into more complex chemical concepts.

Given the importance of the units and measurements chapter and the role of factors-labels in dimensional analysis today, it is reasonable to expect that they have always profoundly influenced the historical development of general chemistry textbooks. Furthermore, it is expected that studies would validate the effectiveness of the factor-label method over alternative unit conversion techniques. However, despite criticisms of the factor-label method (Canagaratna, 1993; Cardulla, 1987), no alternative algorithmic techniques have been proposed as replacements. For instance, Treese and Steven's extensive treatise on units (2018) does not include a section dedicated to algorithmic methods.

Considering this, the present study has two main objectives: first, **to address whether the section dedicated to dimensional analysis has always maintained the same relevance as it does today**, and second, **to examine whether the factor-label method is the only viable methodology for conducting dimensional analysis**, to the point where the two terms are considered synonymous. These questions challenge the authority of modern chemistry textbooks and the presumption that there are no alternative rational approaches

Methodology

Textbook Selection:

We consider the textbook to be a key instrument for assessing the importance of the units and measurements chapter, as it reflects the organization, design, and overall structure of the chemistry course at a given time (Souza & Porto, 2012). For sample collection, a personal database was utilized, which, although useful, had limitations in terms of its temporal depth; finding sources predating the year 2000 was nearly impossible. However, thanks to the Open Library application (Internet Archive, 2024), access to a broader sample was made possible, which we restricted to the period spanning from the 20th century to the present day of the 21st century.

To conduct the analysis of textbooks, specific evaluation criteria were established to address the questions posed and determine the presence and quality of the dimensional analysis section. These criteria include:

(1) The units and measurements section are not included in the text.

(2) The section is present but lacks numerical examples or other graphical paratexts related to quantities.

(3) Calculations such as densities, heats, temperatures are found, but there are no explicit examples of unit conversion; and

(4) The units and measurements chapter is found, with examples of unit conversion using conversion factors. The results of this research are presented in Table 1.

Alternative to the Factor-label method:

Conversion factors are so common that they are considered synonymous with dimensional analysis (Ellis, 2013). The idea of seeking an alternative to them may seem absurd at first glance. In this paper, we present the conclusions of four years of didactic research focusing on the exposition of a series of techniques collectively referred to as **algebraic substitution**. The method of algebraic substitution is based on a simple axiom: the units of measurement of a physical quantity can be considered as algebraic entities, susceptible to direct substitution, apart from being subject to any other algebraic operation.

Results of textbook analysis

No	Reference	Cat.
	1900-1909	
1	McGregory., J. F. (1902)*. Lecture notes on General Chemistry.* Hamilton.	(1)
2	Tillman, S. E. (1907). *Descriptive General Chemistry: A Text-book for Short Course*. J. Wiley & Sons.	(1)
3	Smith, A. (1908)*. General chemistry for colleges*. Century Company.	(1)
	1910-1919	
4	Stoddard, J. T. (1910)*. Introduction to General Chemistry*. Macmillan Company.	(1)
5	Hale., W. J. (1911)*. The calculations if General Chemistry with definitions, explanations, and problems*. D. Van Nostrand Company	(2)
6	Ostwald., W. (1912)*. Outlines of general chemistry*. Macmillan Company.	(1)
7	McPherson, W., & Henderson, W. E. (1913)*. A course in general chemistry*. Ginn.	(1)
8	Newell, L. C. (1914)*. General Chemistry..*. DC Heath & Company.	(1)
9	McPherson, W., & Henderson, W. E. (1915)*. A course in general chemistry*. Ginn.	(1)
10	Cady, H. P. (1916)*. General chemistry*. McGraw-Hill book Company, Incorporated.	(2)
11	Smith, A. (1918)*. General chemistry for colleges*. Century Company.	(1)
12	McCoy., H. N., and Terry., E. M. (1918)*. Introduction to general chemistry*. The University of Chicago press.	(1)
	1920-1929	

TABLE 1. Research Results and Categorization of Examined Textbooks According to the Categories of Section 2.1.

13	Copaux, H. E (1920). *Introduction to General Chemistry: An Exposition of the Principles of Modern Chemistry*. P. Blakiston's son & Company.	(3)
14	McCoy, H. N., & Terry, E. M. (1920). *Introduction to general chemistry*. McGraw-Hill book Company, Incorporated.	(1)
15	Caven R. M. (1920). *The foundations of chemical theory the elements of physical and general chemistry*. D. Van Nostrand Company	(1)
16	McPherson, W., & Henderson, W. E. (1921). *A course in general chemistry*. Ginn and Company.	(1)
17	Caven R. M. (1921). *The foundations of chemical theory the elements of physical and general chemistry*. D. Van Nostrand Company	(1)
	1930-1939	
18	Holmes, H. N. (1930). *General Chemistry*. Macmillan Company.	(1)
19	Holmes, H. N. (1936). *General Chemistry*. Macmillan Company.	(1)
20	White, A. S. (1938). *General science chemistry*. Dent and Sons.	(1)
21	McPherson, W., Henderson, W. E. & Fowler, G. W. (1938). Chemistry at work. Ginn and Company.	(1)
	1940-1949	
22	Young, L. E., & Porter, C. W. (1940). *General Chemistry: A First Course*. Prentice-Hall.	(1)
23	Brinkley, S. R. (1945). Introductory general chemistry. Macmillan Company.	(1)
24	Schoch, p. E. (1946). *General chemistry*. McGraw-Hill book Company, Incorporated.	(1)
25	Pauling, L. (1947). *College chemistry: an introductory descriptive chemistry and modern chemical theory*. W. H. Freeman and Company.	(1)
26	Ray, F. E. (1947). General chemistry. Lippincott.	(3)
27	McPherson, W., Henderson, W. E. & Fowler, G. W. (1948). Chemistry at work. Ginn and Company.	(1)
28	Holmes, H. N. (1949). *General Chemistry*. Macmillan Company.	(3)
29	Sisler, H. H., VanderWerf, C. A., & Davidson, A. W. (1949). General chemistry: a systematic approach. The Macmillan Company.	(1)
	1950-1959	
30	Pauling, L. (1950). *College chemistry: an introductory textbook of general chemistry*. W. H. Freeman and Company.	(2)
31	Selwood, P. W. (1954). *General chemistry*. Henry and Holt Company.	(1)
32	Rochow, E. G., & Wilson, M. K. (1954). General chemistry: a topical introduction.	(1)
33	Pauling, L. (1955). *College chemistry: an introductory textbook of general chemistry*. W. H. Freeman and Company.	(2)
34	Timm, J. A. (1956). *General chemistry*. McGraw-Hill book Company, Incorporated.	(1)
35	Young, L. A., and Porter, C. W. (1958). *General chemistry, a first course*. Prentice-Hall.	(1)

36	Pauling, L. (1959). *College chemistry: an introductory textbook of general chemistry*. W. H. Freeman and Company.	(3)
37	Nebergall, W. H., Schmidt, F. C., & Holtzclaw, H. H. (1959). *General chemistry*. D C Heath and Company	(2)
	1960-1969	
38	Brown, T. L. (1963). *General chemistry*. Charles E. Merril Books.	(1)
39	Pauling, L. (1964). *College chemistry: an introductory textbook of general chemistry*. W. H. Freeman and Company.	(3)
40	Brown, T. L. (1965). *Introduction to general chemistry*. The C. V. Mosby Company.	(4)
41	Holmes, J. K. (1965). *Introduction to general chemistry*. The C. V. Mosby Company.	(4)
42	Nevill, W. A. (1967). *General Chemistry*. McGraw-Hill Book Company.	(4)
43	Burman, H. G. (1968). Principles of general chemistry. *Allyn and Bacon Inc*.	(1)
44	Routh, J. I., Eyman, D. P., and Burton, D. J. (1969). Essentials of general, organic, and biochemistry. *W. B. Saunders Company.*	(3)
	1970-1979	
45	Pauling, L. (1970). *General chemistry*. W. H. Freeman and Company.	(3)
46	Eastman, R. H. (1970). *General chemistry: experiment and theory*. Holt, Rinehart and Winston	(1)
47	Keenan, C. W., and Wood, J. H. (1971). *General college chemistry*. Harper & Row Publishers.	(3)
48	Hammond, G. S., Osteryoung., J., Crawford., T. H., and Gray, H. B. (1971). *Models in chemical science an introduction to general chemistry*. W. A. Benjamin	(3)
49	Jensen, J. T., and Ferren, W. P. (1971). *College general chemistry*. Merrill	(3)
50	Nebergall, W. H., Schmidt, F. C., & Verhoek, F. H. (1972). *Experimental general chemistry*. D C Heath and Company	(3)
51	Borrow, G. M. (1972). *General chemistry*. Wadsworth Pub. Co.	(1)
52	Day, R. A., and Johnson., R. C. (1974). *General chemistry*. Prentice-Hall	(1)
53	Longo, F. R. (1974). *General chemistry: interaction of matter, energy, and man*. McGraw-Hill	(1)
54	Pauling, L., and Pauling, P. (1975). *Chemistry*. W. H. Freeman and Company.	(3)
55	Holmes, J. K., and Krimslay, C. S. (1976). *Introduction to general chemistry*. The C. V. Mosby Company.	(4)
56	Keenan, C. W., Wood, J. H., and Kleinfelter, D. C. (1976). *General college chemistry*. Harper & Row Publishers.	(3)
57	Dickerson, R. W., and Geis, I. (1976). *Chemistry, matter, and the universe an integrated approach to general chemistry*. W. A. Benjamin	(1)
58	Routh, J. I., Eyman, D. P., and Burton, D. J. (1977). Essentials of general, organic, and biochemistry. *W. B. Saunders Company*.	(3)
59	March, J., and Winder, S. (1979). *General chemistry*. Mcmillan publishing co,. Inc.	(4)
	1980-1989	

60	Keenan, C. W., Wood, J. H., and Kleinfelter, D. C. (1980). *General college chemistry*. Harper & Row Publishers.	(4)
61	Nebergall, W. H., Schmidt, F. C., & Holtzclaw, H. H. (1980). *General chemist*ry. D C Heath and Company	(4)
62	James, M. L., Schreck, J. O., and BeMiller, J. N. (1980). *General chemistry*. McGraw-Hill	(4)
63	Siebring, B. R., and Schaff, M. E. (1980). *General chemistry*. Wadsworth Pub. Co.	(4)
64	Rusell, J. B. (1980). *General chemistry*. McGraw-Hill	(4)
65	McQuarrie, D. A., and Rock, P. A. (1984). *General chemistry*. H. H. Freeman and company	(4)
66	Gilleland, M. J. (1982). *Introduction to general, organic, and biological chemistry*. West Pub. Co.	(4)
67	Rusell, J. B. (1980). *General chemistry*. McGraw-Hill	(4)
68	Chang, R. (1984). *Chemistry*. Random House	(4)
69	Nebergall, W. H., Schmidt, F. C., & Holtzclaw, H. H. (1984). *General chemistry*. D C Heath and Company	(4)
70	Witten, K. W. and Gailey, K. D. (1984). *General chemistry*. Saunders college publishing.	(4)
71	McQuarrie., D. A., and Rock., P. A. (1984). *General Chemistry*. W. H. Freeman and Company	(4)
72	McQuarrie., D. A., and Rock., P. A. (1987). *General Chemistry*. W. H. Freeman and Company	(4)
73	Wolfe, D. H. (1986). *Essentials of general, organic, and biological chemistry*. McGraw-Hill	(4)
74	Petrucci, R. H., and Wismer, R. K. (1987). *General chemistry with qualitative analysis*. Macmillan publishing company	(4)
75	McQuarrie., D. A., and Rock., P. A. (1987). *General Chemistry*. W. H. Freeman and Company	(4)
76	Ouellette., R. J. (1988). *Introduction to general, organic, and biological chemistry*. Macmillan, Collier Macmillan	(4)
77	Witten., K. W., and Robinson, W. R (1988). *General chemistry with qualitative analysis*. Saunders college publishing.	(4)
78	Holtzclaw., H. F., Gailey, K. D., and Davis., R. E. (1988). *General chemistry*. D.C. Heath	(4)
79	Brescia., F., Arents, J., Meislich, H., and Turk., A. (1988). *General chemistry*. Harcourt Brace Jovanovich	(4)
80	Atkins, P. W. (1989). *General chemistry*. W. H. Freeman and Company	(4)
	1990-1999	
81	Oxtoby, D. W., and Nachtrieb, N. H. (1990). *Principles of modern chemistry*. Saunders College Publishing.	(1)
82	Bettelheim, F. A., and March, J. (1991). *Introduction to general, organic, and biological chemistry*. Saunders College Publishing. D C Heath & Co	(4)

83	Holtzclaw., H. F., Robinson, W. R., Odom, J. D. (1991). *General chemistry*. D.C. Heath	(4)
84	Witten, K. W., Gailey, K. D., and Davis, R. E (1992). *General chemistry with qualitative analysis*. Saunders college publishing.	(4)
85	Atkins, P. W., and Beran J. A. (1994). *General chemistry*. W. H. Freeman and Company	(4)
86	Oxtoby, D. W., and Nachtrieb, N. H. (1995). *Principles of modern chemistry*. Saunders College Publishing.	(1)
87	Atkins, P. W. (1995). *Chemistry : molecules, matter & change*. W.H. Freeman and Co.	(4)
88	Matta, M. S., Wilbraham, A. C., and Staley, D. D. (1996). *Introduction to general, organic, and biological chemistry*. D. C. Heath and Company.	(4)
89	Witten, K. W., Gailey, K. D., and Davis, R. E (1996). *General chemistry with qualitative analysis*. Saunders college publishing.	(4)
90	Ouellette, R. J. (1997). *Introduction to general, organic, and biological chemistry*. Prentice Hall.	(4)
91	Robinson, W. R., Odom, J. D., and Holtzclaw, H. F. (1997). *Essentials of general chemistry*. Houghton Mifflin Company.	(4)
92	Stoker, H. S. (1998). *General, organic, and biological chemistry*. Houghton Mifflin Company.	(4)
93	Ebbing, D. D., and Gammon, S. D. (1999). *General chemistry: with chemistry study skills*. Houghton Mifflin Company.	(4)
	2000-2009	
94	Blei, I., and Odian, G. (2000). *General, Organic, and Biochemistry*. W. H. Freeman and Company	(4)
95	Chang, R. (2003). *Chemistry*. McGraw-Hill	(4)
96	Ebbing, D. D., and Gammon, S. D. (2002). *General chemistry: with chemistry study skills*. Houghton Mifflin Company.	(4)
97	Glanville, J. O. (2002). *General chemistry for engineers*. Prentice Hall	(4)
98	Bettelheim, F. A., Brown, W. H., and March, J. (2004). *Introduction to general, organic, and biological chemistry*. Thomson Brooks Cole.	(4)
99	Myers, R. T., Oldham, K. B., and Tocci, S. (2004). *Holt chemistry*. Holt, Rinehart, and Wiston.	(4)
100	Moore, J. W., Stanitski, C. L., and Jurs, P. C.. (2005). *Chemistry; the molecular science*. Thomson Brooks Cole.	(4)
101	Ebbing, D. D., and Gammon, S. D. (2005). *General chemistry: with chemistry study skills*. Houghton Mifflin Company.	(4)
102	Blei, I., and Odian, G. (2006). *General, Organic, and Biochemistry*. W. H. Freeman and Company	(4)
103	Whitten, K. W., Davis, R. E., Peck, M. L, and Stanley, G. G. (2006). *General Chemistry The Core*. Brooks Cole	(4)
104	Starzak, M. E. (2006). *Essential General Chemistry*. Pearson Custom Publishing	(4)

105	Brown, T. L., LeMay, H. E., Bursten, B. E., Murphy, C. J., and Wooward, P. M. (2009). *Pearson*. Prentice-Hall.	(4)
106	McMurry, J., Castellion, M. E., and Ballantine, D. S. (2007). *Fundamentals of General, Organic, and Biological Chemistry*. Pearson	(4)
107	Petrucci, R. H., Warwood, W. S., and Herring, F. G. (2007). *General chemistry*. Prentice-Hall	(4)
108	Zumdahl, S. S., and Zumdahl, S. A. (2007). *Chemistry*. Houghton Mifflin Company.	(4)
109	Silberberg, M. S. (2007). *Principles of general chemistry*. McGraw Hill	(4)
110	Ebbing, D. D., and Gammon, S. D. (2008). *General chemistry: with chemistry study skills*. Houghton Mifflin Company.	(4)
111	Brown, T. L., LeMay, H. E., Bursten, B. E., Murphy, C. J., and Wooward, P. M. (2009). *Pearson*. Prentice-Hall.	(4)
112	Silberberg., M. S. (2009). *Chemistry, the Molecular Nature of Matter and Change*. McGraw-Hill	(4)
	2010-2019	
113	Chang, R. (2010). *Chemistry*. McGraw-Hill	(4)
114	Smith, J. G.. (2010). *General, organic, and biological chemistry*. McGraw-Hill	(4)
115	Petrucci, R. H., Herring, F. G., Madura, J. D. and Bissonnette, C. (2010). *General chemistry, principles and Modern Applications*. Pearson Canada	(4)
116	Whitten, K. W., Davis, R. E., Peck, M. L, and Stanley, G. G. (2010). *Chemistry*. Brooks Cole	(4)
117	Chang, R., and Overby, J. (2011). *Chemistry, the essential concepts*. McGraw-Hill	(4)
118	Gallager, R., and Ingram, P. (2011). *Complete chemistry for Cambridge IGCSE*. Oxford University Press	(1)
119	Brown, T. L., LeMay, H. E., Bursten, B. E., Murphy, C. J., and Wooward, P. M. (2012). *Chemistry, the central science*. Prentice-Hall.	(4)
120	Gilbert., T. R., Kirss., R. V., Foster, N., and Davies, G. (2012). *Chemistry, the science in context*. W. W. Norton and Company.	(4)
121	McMurry, J., Fay, R. C., and Fantini, J. (2012). *Fundamentals of General, Organic, and Biological Chemistry*. Prentice-Hall.	(4)
122	Timberlake, K. C. (2012). *Chemistry, an Introduction to General, Organic, and Biological Chemistry*. Prentice-Hall.	(4)
123	Jespersen, N. D., Brady, J. E., and Hyslop, A. (2013). *Chemistry, the molecular nayure of matter*. Wiley.	(4)
124	Silberberg, M. S. (2013). *Principles of general chemistry*. McGraw Hill	(4)
125	Raymond., K. W. (2014). *General, organic, and biological chemistry*. Wiley	(4)
126	Chang, R., and Goldsby, K. A. (2014). *Chemistry*. McGraw-Hill	(4)
127	Brown, T. L., LeMay, H. E., Bursten, B. E., Murphy, C. J., Wooward, P. M., and Stolzfus, M. W. (2015). *Chemistry, the central science*. Pearson.	(4)
128	Burdge., J. and Overby, J. (2015). *Chemistry, the atoms first*. McGraw-Hill.	(4)
129	Rice University. (2015). *Chemistry*. OpenStax College.	(4)

130	Timberlake, K. C. (2015). *Chemistry, an Introduction to General, Organic, and Biological Chemistry*. Prentice-Hall.	(4)
131	Chang, R., and Goldsby, K. A. (2016). *Chemistry*. McGraw-Hill	(4)
132	Brown, T. L., LeMay, H. E., Bursten, B. E., Murphy, C. J., Wooward, P. M., and Stolzfus, M. W. (2017). *Chemistry, the central science Global Edition*. Pearson.	(4)
133	Brown, T. L., LeMay, H. E., Bursten, B. E., Murphy, C. J., Wooward, P. M., Stolzfus, M. W., and Lufaso, M. W. (2017). *Chemistry, the central science*. Pearson.	(4)
134	Ebbing, D. D., and Gammon, S. D. (2018). *General chemistry: with chemistry study skills*. Cengage learning	(4)
135	Zumdahl, S. S., Zumdahl, S. A., and DeCoste, D. J (2018). *Chemistry*. Houghton Mifflin Company.	(4)
136	Chang, R., and Overby, J. (2019). *Chemistry*. McGraw-Hill	(4)
137	Timberlake, K. C. and Orgill, M. (2019). *Chemistry, an Introduction to General, Organic, and Biological Chemistry, Structures of life*. Prentice-Hall.	(4)
	2020-2024	
138	Chang, R., and Overby, J. (2021). *Chemistry*. McGraw-Hill	(4)
139	Brown, T. L., LeMay, H. E., Bursten, B. E., Murphy, C. J., Wooward, P. M., Stolzfus, M. W., and Lufaso, M. W. (2022). *Chemistry, the central science*. Pearson.	(4)
140	Seager, S. L., Slabaugh, M. R., and Hansen, M. S. (2022). *Chemistry for Today: General, Organic, and Biochemistry*. Cengage.	(4)

Using the information from Table 1, we configured a bar chart divided by decades to identify the transition stages between the absence and presence of the Units and Measurements chapter. The details are shown in Figure 1.

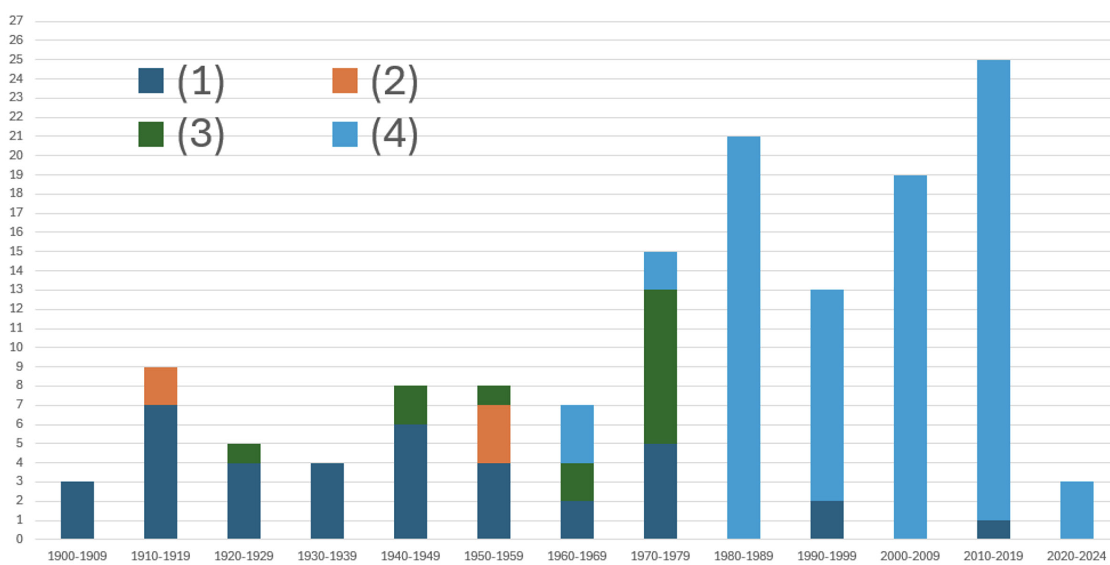

FIGURE 1. The distribution of the 4 analysis categories from Table 1 by decade reveals that the units and measurements chapter has not always been present in chemistry textbooks. Its evolution is primarily observed between the decades of the late 1950s and 1970s. By the end of the 1970s, both the distribution of subtopics and algorithms take on a clearly recognizable form for a modern reader..

Results of the Alternative to the Factor-label method

PREFIX	SYMBOL	MULTIPLIER	EXPONENT FORM
exa	E	1, 000, 000, 000, 000, 000, 000	10^{18}
peta	P	1, 000, 000, 000, 000, 000	10^{15}
tera	T	1, 000, 000, 000, 000	10^{12}
giga	G	1, 000, 000, 000	10^{9}
mega	M	1, 000, 000	10^{6}
kilo	k	1, 000	10^{3}
hecto	h	100	10^{2}
deca	da	10	10^{1}
Basic Unit	Basic Unit	1	10^{0}
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0. 001	10^{-3}
micro	μ	0. 000, 001	10^{-6}
nano	n	0. 000, 000, 001	10^{-9}
pico	p	0. 000, 000, 000, 001	10^{-12}
femto	f	0. 000, 000, 000, 000, 001	10^{-15}
atto	a	0. 000, 000, 000, 000, 000, 001	10^{-18}

FIGURE 2. The decimal prefix table and scientific notation are crucial in the algebraic method, providing algebraic equalities for direct substitutions of units and magnitudes. These prefixes simplify unit conversions, streamlining calculations and integrating dimensional analysis without the need for more complex methods like chemical arithmetic.

Before we begin exploring the techniques, it is important to note that equality tables are fundamental to these methods, with the decimal prefix table being the most relevant. These tables provide a solid foundation for performing conversions and simplifying calculations. Additionally, it should be emphasized that a single exercise can be solved using more than one technique, allowing the operator or student to have multiple analytical routes leading to the same answer. The goal here is to make the mind more flexible, enabling different approaches to be chosen depending on the situation and context, without compromising on the accuracy of the answer.

Substitution of scientific notation by decimal prefixes.

It is generally stated that every number is implicitly accompanied by the scientific notation . Therefore, when we move the decimal separator, we must adjust the exponent accordingly. The exponent becomes positive when the decimal separator moves toward the more significant digits, and negative when it moves toward the less significant digits. In practice, is not explicitly present, but rather summoned through the number of decimal positions the decimal separator is moved by. The idea is that we can **use the decimal system equivalence table (see Figure 2)** to apply the property of **algebraic equality**. In other words, since the prefixes have direct equivalences to certain key scientific notations, we can substitute the scientific notation directly when shifting the decimal position. For instance, moving three positions toward the more significant digits allows us to summon the prefix "kilo" instead of .

Example 1. Saline solution for intravenous infusion is used to treat dehydration and has a number of other important uses in medicine. A partial bag of saline solution has a mass of 600 grams. Express this mass in kilograms and milligrams (Seager, Slabaugh, & Hansen, 2022; Example 1.6).

Factor Label.

Since 1 kg = 1000 g, 600 g can be converted to kilograms as follows:

$$600\ \mathbf{g} \times \frac{1\ \mathbf{kg}}{1000\ \mathbf{g}} = 0.600\ \mathbf{kg}$$

Also, because 1 g = 1000 mg

$$600\ \mathbf{g} \times \frac{1000\ \mathbf{mg}}{1\ \mathbf{g}} = 600\ 000\ \mathbf{mg}$$

Algebraic.

Since the "kilo" prefix represents three positions of the decimal separator to the left:

$$m(\text{salt}) = 600.\ \mathbf{g} = 0.600 \times 10^{3}\ \mathbf{g} = 0.600\ \mathbf{kg}$$

Also, because the "milli" prefix equals three positions to the right:

$$m(\text{salt}) = 600.\ \mathbf{g} = 600\ 000. \times 10^{-3}\ \mathbf{g} = 600\ 000\ \mathbf{mg}$$

Example 2. Convert 1 480 257 g to megagrams and round to two significant figures.

Factor Label.

$$1\ 480\ 257\ \mathbf{g} \times \frac{1\ \mathbf{Mg}}{10^{6}\ \mathbf{g}} = 1.5\ \mathbf{Mg}$$

Algebraic.

$$m_i = 1\ 480\ 257\ \mathbf{g} = 1.480\ 257 \times 10^{6}\ \mathbf{g} = 1.5\ \mathbf{Mg}$$

In the two previous examples in algebraic form, we must keep in mind that the intermediate step of scientific notation (0.600 $\times 10^{3}$ g or 600 000. $\times 10^{-3}$ g) can be done mentally. Once you get used to moving the decimal places, you can replace them with their equivalent decimal prefixes instead of using their equivalent scientific notations.

Linear Prefix substitution

Units of measurement in chemistry can be linear, quadratic, or cubic. In linear substitutions, prefixes can be directly substituted by their scientific notation definitions according to the table of decimal prefixes.

Example 3. Convert 8750 mg to grams.

Factor Label.

$$8750\ \mathbf{mg} \times \frac{1\ \mathbf{g}}{1000\ \mathbf{mg}} = 8.75\ \mathbf{g}$$

Algebraic version 1.

$$m_i = 8.750\ \mathbf{mg} \times 10^{-3} = 8.75\ \mathbf{g}$$

Algebraic version 2.

$$m_i = 8.750\ (10^{-3})\mathbf{g} = 8.75\ \mathbf{g}$$

Prefix and direct substitutions can be done in two ways.

1. The first method involves canceling the unit to be replaced and then placing the equivalent unit as a product. This maintains equality and allows for traceability in dimensional analysis. From experience, this method is used when substitutions are not anticipated in advance and corrections need to be made later in an exercise.

2. The second method involves substituting directly on the unit in a more algebraically correct manner. The advantage of this method is that it is more compact and direct, but at the cost of reduced traceability. From experience, this approach is typically used when dimensional analysis is performed beforehand.

Non-linear Prefix substitution

For non-linear substitution, such as square or cubic units, it is important to note that the power affects the entire composite unit. For example, in the unit cubic centimeter, the cube does not apply solely to the meter; the associated "centi" is also cubed.

Example 4. Convert 2.5 cm^3 to m^3.

Factor Label.

$$2.5\ \mathbf{cm^3} \times \left(\frac{1\ \mathbf{m}}{100\ \mathbf{cm}}\right)^3 = 2.5 \times 10^{-6}\ \mathbf{m^3}$$

Algebraic version 1.

$$V_i = 2.5\ \mathbf{\not{c}m^3} \times (10^{-2})^3 = 2.5 \times 10^{-6}\ \mathbf{m^3}$$

Algebraic version 2.

$$V_i = 2.5\ (10^{-2})^3\mathbf{m^3} = 2.5 \times 10^{-6}\ \mathbf{m^3}$$

The opposite operation, that is, adding a non-linear decimal prefix, is the most complex algebraic operation. This is because it involves more than just moving decimal places according to the prefix definition. In this case, the decimal positions are equal to the prefix definition multiplied by the power of the base unit. Therefore, a different approach is needed. One possible method is using the interpretation of the modulus. Any number multiplied by 1 maintains its definition. The idea is that, if we multiply the prefix by its opposite definition, the result is the modulus. For example, (1 = *kilo* × 10^{-3}; 1 = *mili* × 10^3) Thus, we can state that there is a modulus of 1^3 to the left of the base unit and directly substitute it.

Example 5. Convert 17.2 m^3 to dm^3.

Factor Label.

$$17.2\ \mathbf{m^3} \times \left(\frac{10\ \mathbf{dm}}{1\ \mathbf{m}}\right)^3 = 1.72 \times 10^4\ \mathbf{dm^3}$$

Algebraic version 1.

$$\begin{aligned} r_x = 17.2\ \mathbf{m^3} &= 17.2 \times 1^3\ \mathbf{m^3} \\ &= 17.2 \times (deci \times 10^2)^3\ \mathbf{m^3} \\ &= 1.72 \times 10^4\ \mathbf{dm^3} \end{aligned}$$

That said, since most chemical parameters are linear, this operation is particularly uncommon, except in hydrostatic exercises related to pressure definitions.

Direct Substitution.

In this operation, we interpret the theoretical equality "1 A = y B" as A = y B." Here, "A" and "B" represent two different measurement systems, and "y" is a number other than 1. For example: 1 h = 60 min, which is expressed algebraically as h = 60 min.

Example 6. Convert 8.50 in to cm. Note that 1 in = 2.54 cm.

Factor Label.

$$8.50\ \mathbf{\not{in}} \times \frac{2.54\ \mathbf{cm}}{1\ \mathbf{\not{in}}} = 21.6\ \mathbf{cm}$$

Algebraic version 1

$$r_x = 8.50\ \mathbf{\not{in}} \times 2.54\ \mathbf{cm} = 21.6\ \text{cm}$$

Algebraic version 2

$$r_x = 8.50\ (2.54\ \mathbf{cm}) = 21.6\ \text{cm}$$

Example 7. Convert 85.4 x 10^2 L to m^3. We consider that m^3 = 1000 L.

Factor Label.

$$85.4 \times 10^2\ \mathbf{L} = 8.54 \times 10^3\ \cancel{\mathbf{L}} \times \frac{1\ \mathbf{m^3}}{1000\ \cancel{\mathbf{L}}} = 8.54\ \mathbf{m^3}$$

Algebraic version 1

$$V_i = 85.4 \times 10^2\ \mathbf{L} = 8.54 \cancel{\times 10^3\ \mathbf{L}} \times \mathbf{m^3} = 8.54\ \mathbf{m^3}$$

Algebraic version 2

$$V_i = 85.4 \times 10^2\ \mathbf{L} = 8.54\ (\mathbf{m^3}) = 8.54\ \mathbf{m^3}$$

Inverse Substitution

For inverse substitutions to the rule "A = y B," what we do is isolate the unit "B" by moving "y" to the other side of the equation with a negative exponent " y^{-1} A = B". From that point onward, the process operates the same as with direct substitutions.

Example 8. Convert 30 s to min. Note that 1 min = 60 s.

Factor Label.

$$30\ \cancel{\mathbf{s}} \times \frac{1\ \mathbf{min}}{60\ \cancel{\mathbf{s}}} = 0.50\ \mathbf{min}$$

Algebraic version 1

$$t_i = 30\ \cancel{\mathbf{s}} \times 60^{-1}\ \mathbf{min} = 0.50\ \mathbf{min}$$

Algebraic version 2

$$t_i = 30\ (60^{-1}\ \mathbf{min}) = 0.50\ \mathbf{min}$$

Some units have special interpretations in chemistry when read as conversion factors. Such is the case with "parts per" notation. In the following example, we will present both of its interpretations.

Example 9. Convert 40% by weight to decimal fraction. We assume that $1 = 100\ \% \rightarrow 10^{-2} = \%$

Factor Label 1.

$$40\ \cancel{\%} \times \frac{1}{100\ \cancel{\%}} = 0.40$$

Algebraic version 1

$$w_i = 40\ \cancel{\%} \times 10^{-2} = 0.40$$

Factor Label 2.

$$\frac{40\ \mathbf{g\ sle}}{100\ \mathbf{g\ tot}} = \frac{0.40\ \mathbf{g\ sle}}{1\ \mathbf{g\ tot}}$$

Algebraic version 2

$$w_i = 40\ (10^{-2}) = 0.40$$

Example 10. Convert 87.4 ppm by weight to scientific notation. We assume that $1 = 10^6\ \text{ppm} \rightarrow 10^{-6} = \text{ppm}$

Factor Label 1.

$$87.4\ \cancel{\mathbf{ppm}} \times \frac{1}{10^6\ \cancel{\mathbf{ppm}}} = 8.74 \times 10^{-5}$$

Algebraic version 1

$$w_i = 87.4\ \mathbf{ppm} \times 10^{-6} = 8.74 \times 10^{-5}$$

Factor Label 2.

$$\frac{87.4\ \mathbf{g\ sle}}{10^6\ \mathbf{g\ tot}} = \frac{8.74 \times 10^{-5}\ \mathbf{g\ sle}}{1\ \mathbf{g\ tot}}$$

Algebraic version 2

$$w_i = 87.4\ (10^{-6}) = 8.74 \times 10^{-5}$$

In the previous examples, an important detail emerges that is often overlooked, particularly in the chapter on units and measurements. When discussing the factor-label method, we seemingly group two distinct methods under a single umbrella term. These can be categorized as the "chemical factor label" and the "physical factor label."

The physical factor label is characterized by each term consisting of only two elements: the value and the unit of measurement. In contrast, the chemical factor label includes three elements: the value, the unit, and the identity of the substance. The identity of the substance is specified through its molecular formula, its name, a commercial code, an arbitrary code, or intermediate terms such as solute (*sle*), solvent (*slt*), solution (*sln*), total (*tot*), among others.

Nested Substitution

In nested substitutions, the issue of traceability can become problematic. However, problems where this occurs are relatively rare and are confined to specific questions regarding time magnitude. While there are several complex scenarios in chemistry, as we will see later, there are other tools available to make dimensional analysis more flexible while maintaining traceability in the analysis.

Example 11. Convert 1 second to years. Note that 1 year is equivalent to 365.24 days.

Factor label.

$$1.00\ \mathbf{\not{s}} \times \frac{1\ \mathbf{\not{min}}}{60\ \mathbf{\not{s}}} \times \frac{1\ \mathbf{\not{h}}}{60\ \mathbf{\not{min}}} \times \frac{1\ \mathbf{\not{day}}}{24\ \mathbf{\not{h}}} \times \frac{1\ \mathbf{year}}{365.24\ \mathbf{\not{day}}} = 3.17 \times 10^{-8}\ \mathbf{year}$$

Algebraic version 1

$$t = 1.00\ \mathbf{\not{s}} \times 60^{-1}\ \mathbf{\not{min}} \times 60^{-1}\ \mathbf{\not{h}} \times 24^{-1}\mathbf{\not{day}} \times 365.24^{-1}\mathbf{year} = 3.17 \times 10^{-8}\ \mathbf{year}$$

Algebraic version 2

$$t = 1.00\left(60^{-1}\left(60^{-1}\left(24^{-1}\left(365.24^{-1}\mathbf{year}\right)\right)\right)\right) = 3.17 \times 10^{-8}\ \mathbf{year}$$

Algebraic Sum

In an algebraic sum, the units of the quantities must be the same. If this holds true, the units become the subject of the common factor algebraic operation. This process is commonly related to conservation laws, such as the law of conservation of mass, the law of conservation of energy, or sums of forces.

The negative value can be applied either in the formula or during substitution in most summation laws, such as the conservation of mass or Hess's law. However, it is preferable to apply it during substitution to express summation formulas in their shortest possible form, and therefore, in a more elegant manner. This aligns with the mathematical principle that the shortest form of a theorem is the most desirable.

Division, ratio, or proportion analysis

García-García (2020) proposed that in a theorem where a proportion of similar parameters with equivalent units is generated, the units can be implicitly canceled, as they are ultimately eliminated anyway.

Example 12. When heating 10.0 g of calcium carbonate, 4.4 g of carbon dioxide and 5.6 g of calcium oxide are produced. Does the law of conservation of mass hold?

We interpret mass conservation in its zero-sum form, $0 = \Sigma m_i$, where **initial masses are displayed negatively** and final ones positively. Since in this case the **reactants are the initial masses** and the products are the final ones, the formula is displayed as:

Factor label.

When analyzing a summation using chemical markers, it is common to encounter challenges because the rules of algebra state that we cannot add entities with different units—two pears cannot be added to two apples.

$$x\ \mathbf{g\ tot} = 0 = 4.4\ \mathbf{g\ CO_2} + 5.6\ \mathbf{g\ CaO} - 10\ \mathbf{g\ CaCO_3}$$

However, this problem can be avoided if we argue that the question concerns the total number of entities or in this case the total system mass. By doing so, the specific chemical identity is lost, and the general identity is assumed, allowing the summation to proceed.

$$x\ \mathbf{g\ tot} = 0 = 4.4\ \mathbf{g\ tot} + 5.6\ \mathbf{g\ tot} - 10\ \mathbf{g\ tot}$$

$$x\ \mathbf{g\ tot} = 0 = (4.4 + 5.6 - 10)\ \mathbf{g\ tot}$$

Algebraic.

Using the algebraic method, this problem does not arise because the system's identity is stored as a subscript or within parentheses of the dependent variable.

$$m(\mathbf{total}) = 0 = m_{CO_2} + m_{CaO} + m_{CaCO_3}$$

Meanwhile, the terms of the function are replaced solely by values with units, but without chemical identities from the outset.

$$m(\mathbf{total}) = 0 = (4.4 + 5.6 - 10.0)\ \mathbf{g}$$

Example 13. Calculate the mass fraction of 20 g of NaOH in 500 g of water.

Factor label.

$$\frac{20\ \mathbf{g\ NaOH}}{(20+500)\ \mathbf{g\ tot}} = \frac{0.038\ \mathbf{g\ NaOH}}{1\ \mathbf{g\ tot}}$$

Algebraic.

$$w(\mathbf{NaOH}) = \frac{m_{\mathbf{NaOH}}}{m_{\mathbf{tot}}} = \frac{20}{(20+500)} = 0.038$$

However, something similar can be said regarding similar decimal prefix ratio, even if their base units and physical quantities parameters are different. In this case, we apply the law of modulativity of the product, which states that multiplying a number by a modulus does not alter it. In other words, when we have the same prefix multiplying and dividing, they form a modulus that can be easily added or removed without the need for additional calculations.

Example 14. Convert 791 kg/kL methanol to g/L.

Factor label.

$$\frac{80\ \cancel{\mathbf{kg\ CH_3OH}}}{1\ \cancel{\mathbf{kL\ CH_3OH}}} \times \frac{\cancel{1000}\ \mathbf{g\ CH_3OH}}{1\ \cancel{\mathbf{kg\ CH_3OH}}} \times \frac{1\ \cancel{\mathbf{kL\ CH_3OH}}}{\cancel{1000}\ \mathbf{L\ CH_3OH}} = \frac{80\ \mathbf{g\ CH_3OH}}{1\ \mathbf{L\ CH_3OH}}$$

Algebraic.

$$\rho\,(\mathbf{methanol}) = 80\,\frac{\mathbf{kg}}{\mathbf{kL}} = 80\,\frac{\mathbf{g}}{\mathbf{L}}$$

Example 15. A solution of NaOH with a concentration of 2.7 g per liter convert it to mg/mL.

Factor label.

$$\frac{2.7\ \cancel{\text{g NaOH}}}{1\ \cancel{\text{L } sln}} \times \frac{\cancel{1000}\ \textbf{mg NaOH}}{1\ \cancel{\text{g NaOH}}} \times \frac{1\ \cancel{\text{L } sln}}{\cancel{1000}\ \textbf{mL H}_2\textbf{O}} = \frac{2.7\ \textbf{mg NaOH}}{1\ \textbf{mL H}_2\textbf{O}}$$

Algebraic.

$$\gamma(\text{NaOH}) = 2.7\frac{\text{mg}}{\text{mL}}$$

Prefix transfer

Since decimal prefixes are essentially shorthand for scientific notation, they are subject to all its rules, with the most useful being the commutative property of multiplication— the order of the factors does not alter the product. In simple multiplications, this means that we can transfer a decimal prefix from one base unit to another as needed.

Example 16. A gas occupies 500 mL at 3.0 atm and 300 K. Calculate the amount of substance in millimoles.

Factor label pure form.

$$500\ \cancel{\text{mL gas}} \times \frac{100\ \cancel{\text{mol gas}} \times 1\ \cancel{\text{K gas}}}{8.206\ \cancel{\text{atm gas}} \times 1\ \cancel{\text{L gas}}} \times \frac{3.0\ \cancel{\text{atm gas}}}{300\ \cancel{\text{K gas}}} \times \frac{1\ \cancel{\text{L gas}}}{\cancel{1000}\ \cancel{\text{mL gas}}} \times \frac{\cancel{1000}\ \textbf{mmol gas}}{1\ \cancel{\text{mol gas}}} = 61\ \textbf{mmol gas}$$

Algebraic.

$$n = \frac{P\,V}{R\,T} = \frac{3.0\ \cancel{\text{atm}} * 500\ \text{m}\cancel{\text{L}}}{0.08206\frac{\cancel{\text{atm}}\ \cancel{\text{L}}}{\text{mol}\ \cancel{\text{K}}}\ 300\ \cancel{\text{K}}} = 61\ \textbf{mmol}$$

Factor label combined form.

Step 1: algebraic

$$n = \frac{P\,V}{R\,T} = \frac{3.0\ \cancel{\text{atm}} * 500\ \cancel{\text{mL}}}{0.08206\frac{\cancel{\text{atm}}\ \cancel{\text{L}}}{\text{mol}\ \cancel{\text{K}}}\ 300\ \cancel{\text{K}}} \times \frac{1\ \cancel{\text{L}}}{1000\ \cancel{\text{mL}}} = 0.061\ \textbf{mol}$$

Step 2: factor label

$$0.061\ \cancel{\text{mol gas}} \times \frac{1000\ \textbf{mmol gas}}{1\ \cancel{\text{mol gas}}} = 61\ \textbf{mmol gas}$$

In ratios, it means we can alter its position between numerators and denominators by shifting it according to its opposite definition, such as kilo vs milli.

Example 17. A solution of 40.0 g/mL NaCl, convert the expression to kg/L.

Factor label.

$$\frac{40.0\ \cancel{\text{g NaOH}}}{1\ \cancel{\text{mL } sln}} \times \frac{1\ \textbf{kg NaOH}}{1000\ \cancel{\text{g NaOH}}} \times \frac{1000\ \cancel{\text{mL } sln}}{1\ \textbf{L H}_2\textbf{O}} = \frac{40.0\ \textbf{kg NaOH}}{1\ \textbf{L H}_2\textbf{O}}$$

Algebraic.

$$(kilo = mili^{-1})$$

$$\gamma(\text{NaOH}) = 4.0\frac{\text{g}}{\text{mL}} = 4.0\frac{\text{kg}}{\text{L}}$$

Although this modifies the profound meaning of a chemical or physical system, as algebraically 1 N = 1 kg m/ s^2 = 1 g km/s^2, the two equalities describe different but equivalent physical-experimental processes. Nevertheless, at the end of the entire chemical or physical process, in the final dimensional analysis, it may be useful to apply the commutative law of multiplication to avoid performing additional calculations that ultimately prove unnecessary.

Product of Opposite Decimal Prefixes:

In such cases, these prefixes can cancel each other directly, without needing arithmetic substitution. For example, the product of the prefixes "kilo" (10^3) and "milli" (10^{-3}) results in a modulus of 1, which simplifies the process.

Example 18. The density of **mercury** is 13.6 g/mL. What is the volume in liters if the mass is measured as 3.5 kg?

Factor Label

$$3.5\ \text{kg Hg} \times \frac{1000\ \text{g Hg}}{1\ \text{kg Hg}} \times \frac{1\ \text{mL Hg}}{13.6\ \text{g Hg}} \times \frac{1\ \textbf{L Hg}}{1000\ \text{mL Hg}} = 0.26\ \textbf{L Hg}$$

Algebraic.

$$V(\textbf{Hg}) = m_{\textbf{Hg}} \cdot \frac{1}{\rho_{\textbf{Hg}}} = 3.5\ \text{kg} \times \frac{\text{m}\textbf{L}}{13.6\ \text{g}} = 0.26\ \textbf{L}$$

Pure Dimensional Analysis vs. Concrete Independent Value Analysis:

When a theorem or law is very complex or needs to be applied repeatedly, such as when expressing the results of a measurement in a data table, it is more efficient to separate the pure dimensional analysis from the value analysis. In pure dimensional analysis, we only substitute the units of each physical parameter to identify their cancellation and to identify dimensional analysis constants. Once we know the units of the answer, we can perform trivial value calculations and place them in data tables, focusing only on the value analysis. This type of operation is particularly relevant when tabulating experimental results, where we need to process many data points related to a single specific theorem (see Example 19).

When carrying out all these procedures, we must always follow a rule: the result must always have a prefix associated with a base unit; otherwise, the dimensional analysis is incorrect, or a different notation should be used. For example, a given value could not be expressed as "micro," but we can express it as "ppm," although in both cases, what we are doing is replacing the power of 10^{-6}.

Analysis

Although we began this investigation focusing solely on two aspects—the presence or absence of the chapter on units and measurements, and the possibility of rethinking chemical mathematics from arithmetic to algebraic rules—we also encountered other interesting elements worth discussing. These include the way chemistry textbooks have been visually represented throughout the 20th century and into the 21st, as well as certain characteristics of the factor-label method that are often overlooked or learned intuitively without a formal discussion of their properties or rationale.

Example 19. The following urea solutions were prepared to create a calibration curve: 1.0 g, 1.5 g, 2.0 g, 2.5 g, 3.0 g, 3.5 g, 4.0 g, and 4.5 g in 200.0 g of water. Express the molality of each case.

Factor Label

$$\frac{1.0\ \text{g urea}}{200.0\ \text{g H}_2\text{O}} \times \frac{1\ \textbf{mol urea}}{60.02\ \text{g urea}} \times \frac{10^3\ \text{g H}_2\text{O}}{1\ \textbf{kg H}_2\textbf{O}} = \frac{0.083\ \textbf{mol urea}}{1\ \textbf{kg H}_2\textbf{O}}$$

$$\frac{1.5\ \text{g urea}}{200.0\ \text{g H}_2\text{O}} \times \frac{1\ \textbf{mol urea}}{60.02\ \text{g urea}} \times \frac{10^3\ \text{g H}_2\text{O}}{1\ \textbf{kg H}_2\textbf{O}} = \frac{0.125\ \textbf{mol urea}}{1\ \textbf{kg H}_2\textbf{O}}$$

$$\frac{2.0\ \text{g urea}}{200.0\ \text{g H}_2\text{O}} \times \frac{1\ \textbf{mol urea}}{60.02\ \text{g urea}} \times \frac{10^3\ \text{g H}_2\text{O}}{1\ \textbf{kg H}_2\textbf{O}} = \frac{0.167\ \textbf{mol urea}}{1\ \textbf{kg H}_2\textbf{O}}$$

$$\frac{2.5\ \text{g urea}}{200.0\ \text{g H}_2\text{O}} \times \frac{1\ \textbf{mol urea}}{60.02\ \text{g urea}} \times \frac{10^3\ \text{g H}_2\text{O}}{1\ \textbf{kg H}_2\textbf{O}} = \frac{0.208\ \textbf{mol urea}}{1\ \textbf{kg H}_2\textbf{O}}$$

$$\frac{3.0\ \text{g urea}}{200.0\ \text{g H}_2\text{O}} \times \frac{1\ \textbf{mol urea}}{60.02\ \text{g urea}} \times \frac{10^3\ \text{g H}_2\text{O}}{1\ \textbf{kg H}_2\textbf{O}} = \frac{0.250\ \textbf{mol urea}}{1\ \textbf{kg H}_2\textbf{O}}$$

$$\frac{3.5\ \text{g urea}}{200.0\ \text{g H}_2\text{O}} \times \frac{1\ \textbf{mol urea}}{60.02\ \text{g urea}} \times \frac{10^3\ \text{g H}_2\text{O}}{1\ \textbf{kg H}_2\textbf{O}} = \frac{0.292\ \textbf{mol urea}}{1\ \textbf{kg H}_2\textbf{O}}$$

$$\frac{4.0\ \text{g urea}}{200.0\ \text{g H}_2\text{O}} \times \frac{1\ \textbf{mol urea}}{60.02\ \text{g urea}} \times \frac{10^3\ \text{g H}_2\text{O}}{1\ \textbf{kg H}_2\textbf{O}} = \frac{0.333\ \textbf{mol urea}}{1\ \textbf{kg H}_2\textbf{O}}$$

$$\frac{4.5\ \text{g urea}}{200.0\ \text{g H}_2\text{O}} \times \frac{1\ \textbf{mol urea}}{60.02\ \text{g urea}} \times \frac{10^3\ \text{g H}_2\text{O}}{1\ \textbf{kg H}_2\textbf{O}} = \frac{0.375\ \textbf{mol urea}}{1\ \textbf{kg H}_2\textbf{O}}$$

Algebraic.

Step 1

Theorem proof: We solve the molal concentration definition substituting the solute amount of substance by the mass to molar mass ratio.

$$b_i = \frac{n_i}{m_j} \quad \Rightarrow \quad b_i = \frac{m_i}{m_j \cdot M_i}$$

Step 2

Dimensional Analysis: In this case, the natural cancellation of units gives us mol/g. However, since molality is expressed in terms of kilograms, we isolate g in (10^3 g = kg) and substitute (g = 10^{-3} kg) into the dimensional analysis.

$$b_i = \frac{m_i}{m_j \cdot M_i} = \frac{\textbf{g}}{10^{-3}\ \textbf{kg} \cdot \frac{\textbf{g}}{\textbf{mol}}} = \frac{\textbf{mol}}{\textbf{kg}}$$

Once the dimensional constants appear, they can be incorporated into the **trivial algorithm**:

$$b(\text{urea}) = m_{\text{urea}}(\textbf{g}) \div (m_{\text{water}}(\textbf{g}) \times 10^{-3} \times M_{urea}(\textbf{g/mol}))$$

Which is the instruction we give to the system, whether it's on a calculator or in Excel, to drag and compute the series of data.

B2 | fx =A2/(200*(10^-3)*60.02)

	A	B
1	*m* (g)	*b* (mol/kg)
2	1.0	0.083
3	1.5	0.125
4	2.0	0.167
5	2.5	0.208
6	3.0	0.250
7	3.5	0.292
8	4.0	0.333
9	4.5	0.375

The units and measures chapter

Despite its modern importance, the chapter on units and measurements has not been a fundamental part of the general chemistry textbook since the early 20th century. On the contrary, its inclusion took place during a transition period spanning from the late 1950s to the late 1970s (see Figure 1 and Table 1), coinciding with other significant changes such as advancements in printing technologies and the distribution of information through paratexts.

The fact that the chapter on units and measurements was not present from the beginning implies that its inclusion was a conscious act suggesting it was also arbitrary, as nowhere is it stated that the factor-label method is the definitive solution as opposed to any other alternative, if one had been considered. However, the fact that the factor-label method has been present from the beginning alongside the dimensional analysis section has led to equating both concepts. However, this equating would only be valid if there were no other alternatives available. In section 4, we examine how it is possible to consider a set of algorithmic techniques, i.e., a series of ordered and systematic steps that lead to a correct solution, which works just as well, at least from a purely theoretical point of view. The aim of this article is not to judge which technique is better or worse, but simply to consider the alternative. By talking about "alternative," **we imply that dimensional analysis and factor-label method are not synonymous**. Dimensional analysis is a broader epistemic category, which ultimately seeks to evaluate the dimensional equality of units between a solved or dependent variable and a statement requirement, compared to a specific mathematical function, such as a formula, law, axiom, or theorem. To achieve this, various techniques can be used; the fact that the most well-known one is the factor-label method does not imply it is the only possible one. Therefore, **we consider reducing the concept of dimensional analysis to a specific technique would be invalid**.

The role of algebraic substitution

According to DeToma (1994), "*algebraic formulations and manipulations of this type are standard practice in modern general chemistry textbooks when treating all quantitative areas (thermodynamics, equilibrium, kinetics, electrochemistry, colligative properties, quantum theory, etc.) except stoichiometry.*" The current trend, as deToma notes, is to treat "*basic problems in chemical composition and reaction stoichiometry as a separate class to be done exclusively by factor-label methods*." While the bulk of the text uses symbolic algebra formulation and manipulation, deToma highlights that material on stoichiometry is handled differently, which he describes as "***inconsistent and unnecessary***."

Building on this argument, we propose that several topics traditionally formalized using algebraic techniques still employ a mixed language with chemical arithmetic. This approach forces students to be flexible in their reading, requiring additional effort to reconcile mathematical calculations with chemical markers. As a result, students are often compelled to break down certain problems into stages: one based on the factor-label method and another involving algebraic techniques (see **Example 16**: **factor label combined form**). While deToma focuses on stoichiometry, we extend this discussion by arguing that the chapter on units and measurements could also benefit from a rethinking of its approach. Such a reframing would allow for a more coherent and accessible treatment of these topics, promoting a more integrative approach between mathematics and chemistry.

It is evident that the algebraic substitution technique is more flexible than the factor-label method, as it offers a greater number of solutions for specific situations, which we could call shortcuts. However, we are aware that this greater flexibility constitutes both its greatest strength and its greatest weakness since it requires a longer instruction and adaptation time. However, once mastered, it allows for more effective dimensional analysis in both chemistry and physics scenarios, which could result in greater confidence when tackling pencil-and-paper exercises.

The question now is: *What would be the practical applicability of algebraic substitution in real educational contexts?* Regarding this, it is worth asking what we mean by *practical applicability,* as its interpretation may vary depending on the perspective. It could refer to the need for greater epistemic coherence, enhanced mathematical consistency, or simply the mechanization of procedures to carry out trivial calculations.

Regarding epistemic coherence, we can distinguish between the concept of *dimensional analysis* and the techniques that allow its application in specific situations—*algorithms.* The former refers to the pure concept, which should ideally be the focus, or at least something we should dedicate more time to. The latter, on the other hand, consists of merely mechanical procedural chains. Personally, I believe we sometimes lose sight of the difference between concepts and their techniques, which can lead to careless arguments. In this sense, the goal of optimizing the mathematical methods in chemistry lies in enabling a sharper focus on the underlying chemical concepts, ensuring that the mechanical aspect becomes more streamlined and less burdensome.

Mathematical consistency is a crucial aspect, as one of the core principles of scientific thinking is to avoid "leaps of faith," particularly when it comes to numerical calculations. As students, we sometimes feel that algorithms are flawed because, in certain scenarios, dimensional analysis using conversion factors does not operate optimally. And if it does, it often relies on intuitive rules that are not explicitly stated.

A classic example of this can be seen in **Example 12** with the summation laws. If we use chemical markers associated with the units of measurement, we encounter a situation where summation cannot proceed because the homogeneity of terms required for algebraic addition is not satisfied—unless we apply the auxiliary rule discussed in that context.

Algebraic techniques, however, tend to be more consistent because most of their rules are already taught in basic algebra courses. This reduces instructional time and avoids the need for special cases unique to chemical arithmetic. Instead, we can focus directly on examples and practice, making the process more efficient and intuitive for students.

Regarding mechanization, although this document does not yet present the results of our classroom research, **we can state that most students tend to favor the algorithm with the least symbolic load**. Fewer symbols to write mean fewer opportunities to make mistakes, making such methods more appealing to learners.

Additionally, algebraic techniques for dimensional analysis allow the direct resolution of special exercises. As deToma mentions, the language of chemical mathematics oscillates between chemical arithmetic ("factor labels") and chemical algebra (the "use of laws and theorems"). Certain special exercises require the interaction of chapters with distinct mathematical approaches, leading to a two-stage resolution process where each stage uses a different mathematical language. This can introduce errors or unnecessarily lengthen the resolution process (see **Example 16**).

One practical use, in terms of mechanical operations, is the tabulation of repetitive data through a standardized process, that is, the formula or conversion factor as a bridge to experimental design. As seen in **Example 19**, while conversion factors constrain us to mechanically repeat the same trivial processes, algebra frees us from this due to its inherent nature: **generalization**. This allows us to translate from the language of human algebra to a formula that can easily be converted into the language of machines, such as Excel, thus accelerating calculations without sacrificing the formality of dimensional analysis.

Factor labels can also be used to model gases and other topics.

In the same **Example 16**, it is also evident that gas chemistry, traditionally solved through algebra (using laws and theorems), can also be addressed using chemical conversion factors. This raises another set of questions: why are some chapters resolved through theorems and others through conversion factors, given that either language can be adapted?

There is no methodological reason why gas-related problems could not be solved using chemical factor labels. A tentative answer would be that solving it using factor labels is longer and more confusing. However, the same can be said about the stoichiometry chapter, where algebraic alternatives have been proposed since the 1970s (García García, 2020; Garst, 1974; Mousavi, 2018). This is contradictory, and therefore, the only hypothesis we can propose to explain this contradiction lies in the inertia of tradition and authority. Textbooks change relatively little and are slow to incorporate alternative ways of thinking. This is as true for cutting-edge science as it is for alternative problem-solving techniques.

This lack of alternative proposals in chemistry textbooks is a telling reflection of how authority in the field, often reinforced by the repetition of established methods, shapes perceptions of what is acceptable or even possible in chemistry education. As Simon (2016) argues, "*what matters here is that the major features of a textbook are its use in teaching and learning, and its* authority." This repeated reliance on the factor-label method suggests that its authority has become so ingrained that it often goes unquestioned, thus stifling potential alternatives. This reflects Owen Hannaway's (1975) observation that the making of chemistry has, in large part, been shaped in classrooms, textbooks, and didactic traditions. In this context, the dominance of the factor-label method may be more about its historical authority than its inherent superiority, leading to the impression that no other rational alternatives exist.

Perhaps the chapters where chemical factor labels are limited are those involving problems with differentials (e.g., chemical kinetics) or quadratic equations (e.g., equilibria). So, why not accustom us to algebraic methods from the start, given that sooner or later, they become the only language adaptable to moderately advanced chemistry?

Chemical identity in factor labels

Factor labels can be divided into two general types: those that focus solely on values and units of measurement, and those that include not only value and unit but also chemical identity. Chemical identity is described in one of three ways: the substance's name, its molecular formula, or codes (either commercial or specific to certain contexts), which include details such as totality, dissolution, solute, and solvent. In this way, there are chemical factor labels and physical factor labels, depending on whether the chemical identity is or is not considered important.

Chemical conversion factors and their specific rules—such as the inability to cancel similar units due to their association with different identities, the substitution of a particular identity with the total in summation laws (e.g., conservation of mass, charge, energy), or the need to interpret or translate a unit in algebraic form within problems into its expanded form as a chemical conversion factor—are not explicitly addressed in the units and measurements chapter. Instead, these rules are learned progressively as these laws are discussed throughout the chemistry chapters.

Example 20. Algebraic interpretation for 0.25 molal of NaCl in water:	**Example 21.** Chemistry factor label interpretation for 0.25 molal of NaCl in water:
$b(\textbf{NaCl: water}) = 0.25\ \mathbf{m} = 0.25\frac{\mathbf{mol}}{\mathbf{kg}}$	$0.25\ \mathbf{m}\ \textbf{NaCl} = \frac{0.25\ \mathbf{mol}\ \textbf{NaCl}}{1\ \mathbf{kg}\ \textbf{water}}$

In the previous examples, we used an inherently chemical unit, molality. To further clarify the differences between the two types of factor labels, we will now present an example involving a unit that can be both physical and chemical: density.

Example 22. Algebraic interpretation 0.9945 g/mL of water:	**Example 23.** Chemical factor label interpretation 0.9945 g/mL of water:	**Example 24.** Physical factor label interpretation 0.9945 g/mL of water:
$\rho(\textbf{water}) = 0.9945\frac{\mathbf{g}}{\mathbf{mL}}$	$\frac{0.9945\ \mathbf{g}\ \textbf{water}}{1\ \mathbf{mL}\ \textbf{water}}$	$\frac{0.9945\ \mathbf{g}}{1\ \mathbf{mL}}$

When analyzing how factor labels are presented in the *Units and Measures* chapter of any modern textbook, you will find that they rely on the physical interpretation, disregarding the identity of the substance. However, in more advanced chapters, where conversions between mass and volume of liquids are required, substance-specific factors will force us to choose between two options: (1) solving the problem in independent steps or sections, or (2) inferring the chemical identity within the conversion factor, which allows its nesting.

With the introduction of textbook paratexts, pencil-and-paper exercises also emerge, which can be distinguished in two stages. In the analysis categories, we see category (3), where processes of calculations involving units and measurements are described. These calculations, such as those for densities, are characterized by an algebraic nature, and lack of chemical identity markers. The value and unit of measurement are simply substituted. Later, in stage (4), we see the introduction of unit conversions using factor-labels, but in most cases, **the chemical identity still does not appear until more advanced chapters**.

From our perspective, if we are going to embrace the factor label method as the gold standard, we should do so correctly. For a chemistry textbook, this means getting accustomed to applying it under the chemical interpretation, where each term includes three elements: **value**, **unit**, and **identity**. These elements are crucial in chapters such as stoichiometry, concentration units, and colligative properties.

Chemical identity in algebraic formulas

Chemical identity is not an issue for the algebraic methodology if the measured parameter is explicitly defined in the dependent variable. This is the reason why, in all examples of algebraic forms, we define the dependent variable, as it includes the chemical identity if stated in the problem. Otherwise, an undefined identity is indicated with the subscript "i.".

Consequently, the dependent variable will consist of more than one symbol, with the two essential components being the primary symbol of the physical quantity (e.g., mass, time, amount of substance) and the chemical identity symbol, represented as a subscript or within parentheses. Additionally, there are subsidiary symbols that convey further information, such as differences (Δ), differentials (δ), initial values ($_o$), standard values provided in tables (°), and even other data like time or temperature conditions, which conversion factors alone are incapable of representing. For example, using algebraic symbols, we can write the standard (°) enthalpy (H) of formation ($_f$) difference (Δ) of NaCl as $\Delta H^{o}f(NaCl)$ among other parameters with complex symbolism that are relatively difficult to describe using only factor labels.

In other words, the dependent or isolated variables in algebraic language allow us to interpret more than just the physical magnitude and the identity of the substance; they enable us to determine whether we are dealing with specific data points, discrete processes, or continuous processes.

Interdisciplinarity between chemistry, mathematics, and physics

Instructors of General Chemistry 1 frequently encounter the challenge of dedicating the first month, or even the entirety of the course, to teaching mathematical concepts rather than focusing on chemical principles. This issue arises from the need to introduce mathematical tools, such as the factor-label method, which many students have not adequately mastered beforehand. While Hoile (1974) suggests that "*the concepts of the factor label method [should be presented] prior to high school chemistry and physics courses within an integrated mathematics/science lesson, drawing applications from the realm of science and justification from the realm of mathematics*," this paper argues for a different approach. Rather than requiring chemistry instructors to teach factor labels or expecting mathematics instructors to integrate chemical arithmetic, this study proposes leveraging the existing arithmetic and algebra skills taught in mathematics courses. This approach removes the need for a specialized mathematical framework, such as **chemical arithmetic "factor labels"**, tailored to a small portion of the chemistry curriculum. Instead, it would allow instructors to focus more effectively on **chemical concepts**, relying on the existing arithmetic and algebra skills already taught in mathematics courses.

Following deToma's reasoning, it is evident that chapters not analyzed through factor labels are addressed using algebra, that is, through equations, whether these are laws or derived theorems. The use of equations is intrinsic to both mathematics and physics, which is why adopting a more algebraic approach in chemistry would enhance conceptual and procedural coherence in the mathematical language, without disregarding the differences between physical and chemical laws. The main distinction between a chemical law or theorem and a physical one lies in the importance of substance identity. This brings us back to the topic of fundamental concepts: one of the major focuses of the epistemology of chemistry is the study of substance identity and its properties. Therefore, theorems in chemistry must include clear indications of substance identity, whether explicit or indeterminate.

Presentation of Chemistry Content in Textbooks

In **Figure 3**, we take a small sample of how chemistry textbooks are visualized. Here, we can clearly see an evolution from a chemistry focused on text-heavy explanations to one distributed among text, paratexts, pen-and-paper exercises, and vibrant, schematic colors designed to break monotony and capture attention.

The production of a textbook involves the work of print technicians and the marketing strategies of publishers whose actions are often not limited to providing a physical container and a shop window, but have a direct impact on the shaping of knowledge (Simon, 2016). When examining earlier texts, or even those from the transitional period, we observe a subjective yet evident reality: they are overwhelmingly flat, with perhaps one or two tables and black-and-white figures (See **Figure 3**). While this is partly due to the printing technologies of the time, it also relates to the way texts were visualized.

Even in black and white, there are ways to break the continuity of the text, such as using boxes, glosses, or highlighted sections. However, these alternatives were not present in textbooks from the first half of the 20th century. Therefore, while printing technology and market competition are important factors, there must be another explanation for this phenomenon.

Although it is not a primary objective of this research, we have noticed a shift in the visualization of chemistry textbooks that coincides with the emergence of pencil-and-paper exercises related to units and measurements. This transition towards greater visual structure and graphic organization in the texts could be influenced by the artistic trends of the second half of the 20th century, such as Abstract Expressionism, Pop Art, and Minimalism. These artistic movements promoted a break from traditional formats, simplification, and the use of striking visual elements, which may have impacted how information is presented in chemistry books. This hypothesis should be further explored in future studies.

Conclusions

1. **Units and Measurements Chapter**: Their inclusion in chemistry books was a gradual shift between the 1950s and 1970s, aligning with technological advancements and the spread of information. However, other methods for dimensional analysis could be explored.

2. **Algebraic Substitution**: Algebraic substitution is more flexible than the factor label method. Despite its initial apparent complexity, it's more effective for advanced problems.

3. **Epistemological Consistency**: It's important to distinguish between dimensional analysis concepts and the methods used to apply them.

4. **Mathematical Consistency**: Algebraic methods tend to be more consistent and are taught in basic algebra courses, which can reduce instructional time and increase efficiency.

5. **Chemical Identity**: The use of factor labels in chemistry textbooks, particularly in the chapter on Units and Measures, should emphasize chemical identity, distinguishing it from similar chapters in physics textbooks. This approach is essential as it is a skill required in later sections of the course.

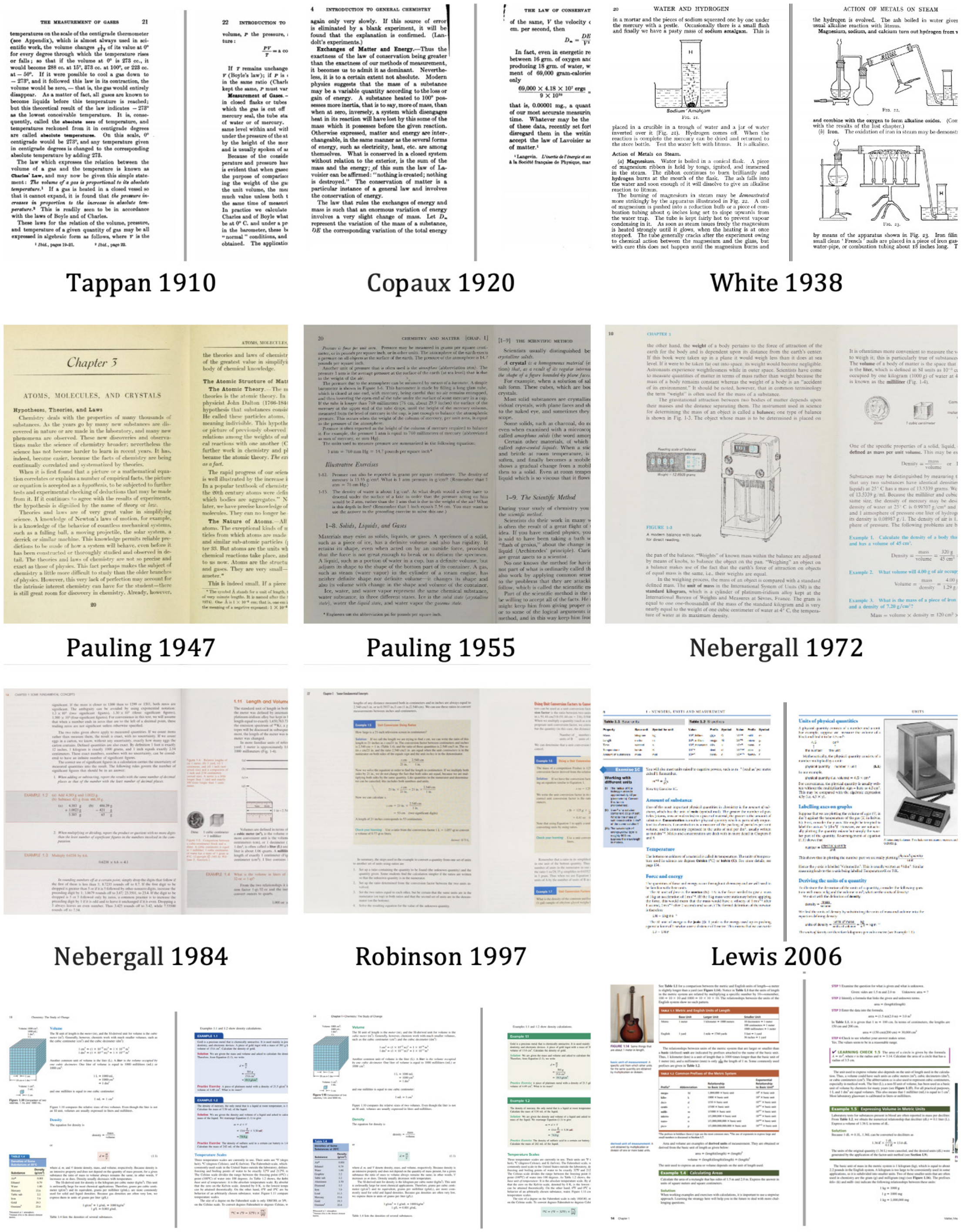

FIGURE 3. Images of a limited sample of the textbooks analyzed in Table 1, showcasing the visualization of information and its evolution throughout the 20th century.

Content Presentation: Over time, chemistry books have evolved visually, incorporating more graphics and diagrams, reflecting artistic movements and advancements in printing technology.

Once these algorithms are outlined, the goal for future studies will be their implementation in the classroom, starting with the assessment of their acceptability by students in quasi-experimental studies, and subsequently, comparing their relative effectiveness in experimental studies.

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