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SIMPLE MODEL WITH TIME-VARYING FINE STRUCTURE "CONSTANT"-PART II

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RESUMEN

Extendiendo la Hipótesis de los Grandes Números de Dirac para cubrir las "constantes" cosmológica y de estructura fina, y la rotación del Universo, las estudiamos, incluyendo una "demonstración", del cálculo de la velocidad angular del Universo actual, y del inflacionario. Se clarifican algunos puntos criticables de este artículo.

ABSTRACT

The extension of Dirac's LNH to cover cosmological and fine-structure time-varying "constants", and the rotation of the universe, is here analysed, including a "derivation" of the angular speed of the present universe, and of the inflationary phase. Controversial points in the present calculation are clarified.

Key Words: cosmological parameters — cosmology: theory — cosmology: miscellaneous — early universe — gravitation

1. INTRODUCTION

We shall study a generalisation of Dirac's LNH universe, with the introduction of time-varying speed of light, which causes a time-varying fine-structure "constant", and a possible rotation of the universe, either for the present time, or for inflationary periods. This paper is a sequel to a previous one dealing with the more or less equivalent consequences of a time-varying electric and magnetic permittivity (Berman 2009).

The rotation of the universe (de Sabbata & Sivaram 1994; de Sabbata & Gasperini 1979) may have been detected experimentally by NASA scientists who tracked the Pioneer probes, finding an anomalous deceleration that affected the spaceships during the thirty years that they took to leave the Solar system. This acceleration can be explained through the rotation of the Machian universe (Berman 2007c). A universal spin has been considered by Berman (2008b,c).

A time-varying gravitational constant, as well as others, was conceived by P.A.M. Dirac (1938, 1974), Eddington (1933, 1935, 1939), Barrow (1990) through his Large Number Hypothesis. Later, Berman supplied the GLNH – Generalised Large Number Hypothesis (Berman 1992a,b, 1994). This hypothesis arose from the fact that certain relationships among physical quantities revealed extraordinary large numbers of the order 10^{40}. Such numbers, instead of being coincidental and far from usual values, were attributed to time-varying quantities, related to the growing number of nucleons in the universe. In fact, the number N, for the present universe, is estimated as $\left(10^{40}\right)^2$. The number is "large" because the universe is "old". At least, this was and still is the best explanation at our disposal.

The four relations below represent, respectively, the ratios among the scalar length of the causally related universe, and the classical electronic radius; the ratio between the electrostatic and gravitational forces between a proton and an electron; the mass of the universe divided by the mass of a proton or a nucleon; and a relation involving the cosmological constant and the masses of neutron and electron.

If we denote the Hubble's constant by$H$, the electron charge and mass by e and m_e, the proton mass by m_p, the cosmological constant by Λ, the speed of light by c, and Planck's constant by h, we have:

$$\frac{cH^{-1}}{(\frac{e^2}{m_e c^2})} \cong \sqrt{N}\,, \tag{1}$$

$$\frac{e^2}{G m_p m_e} \cong \sqrt{N}\,, \tag{2}$$

$$\frac{\rho (cH^{-1})^3}{m_p} \cong \quad N\,, \tag{3}$$

$$ch(m_p m_e/\Lambda)^{1/2} \cong \sqrt{N}\,. \tag{4}$$

We may in general have time-varying speed of light $c = c(t)$; of $\Lambda = \Lambda(t)$; of $G = G(t)$; etc. We define the fine structure "constant" as

$$\alpha \equiv \frac{e^2}{\bar{h}\ c(t)}\,, \tag{5}$$

and consider $\alpha = \alpha(t)$, because of the time-varying speed of light.

2. POWER-LAW VARIATIONS

One can ask whether the previous Section's constant-variations could be caused by a time-varying speed of light: $c = c(t)$. We refer to Berman (2007a) for information on the experimental time variability of α. Gomide (1976) has studied $c(t)$ and α in such a case, which was later revived by Barrow (1998a,b,1997); Barrow & Magueijo (1999); Albrecht & Magueijo (1999); Bekenstein (1982). This could explain also the supernovae observations. We refer to their papers for further information. Our framework now will be an estimate made through Berman's GLNH.

We express now Webb et al.'s (1999, 2001) experimental result as:

$$\left(\frac{\dot{\alpha}}{\alpha}\right)_{\rm exp} \simeq -1.1 \times 10^{-5} t^{-1}\,, \tag{6}$$

where t represents the age of the universe.

From equation (5) we find:

$$\frac{\dot{\alpha}}{\alpha} = -\frac{\dot{c}}{c}\,. \tag{7}$$

Again, we suppose that the speed of light varies with a power law of time:

$$c = At^n \qquad (A = \text{constant})\,. \tag{8}$$

From the above experimental value we find:

$$n \approx 10^{-5}\,. \tag{9}$$

From equations (8) and (9), considering (7), we find:

$$\frac{\dot{\alpha}}{\alpha} = -\frac{\dot{c}}{c} = nt^{-1}\,. \tag{10}$$

From relations (1), (2), (3) and (4) we find:

$$N \propto t^{2+6n}\,, \tag{11}$$

$$G \propto t^{-1-3n}\,, \tag{12}$$

$$\Lambda \propto t^{-2-4n}\,. \tag{13}$$

$$\rho \propto t^{-1+3n}\,. \tag{14}$$

We see that the speed of light varies slowly with the age of the universe. For the numerical value (9), we would obtain:

$$N \propto t^{2.0001}\,, \tag{15}$$

and then:

$$G \propto t^{-1.00005}\,, \tag{16}$$

$$\Lambda \propto t^{-2.0001}\,, \tag{17}$$

$$\rho \propto t^{-0.99995}\,. \tag{18}$$

This is our solution, based on Berman's GLNH, itself based on Dirac's work (Dirac 1938, 1974). A pre-print with a preliminary but incomplete solution was already prepared by Berman & Trevisan (2001a,b,c).

As a bonus we found possible laws of variation for N, G, ρ, and Λ. The Λ-term time variation is also very close and even, practically indistinguishable, from the law of variation $\Lambda \propto t^{-2}$.

It is clear that in this section's model, the electric permittivity of the vacuum, along with its magnetic permeability, and also Planck's constant are really constant here. We point out again, that in the long run, it will be only when a superunification theory becomes available that all but one of the different models offerred in the literature could be discarded (hopefully).

3. EXPONENTIAL INFLATION

Remembering that relations (1) and (3) carry the radius of the causally related universe, cH^{-1}, we substitute it by the exponential relation,

$$R = R_0 e^{Ht}\,. \tag{19}$$

With the same arguments above, but substituting, (8) by the following one,

$$c = c_0 e^{\gamma t}\,, \qquad (c_0, \gamma = \text{constants}) \tag{20}$$

we would find:

$$N \propto e^{[H+2\gamma]\ t}\,, \tag{21}$$

$$G \propto e^{-\left[\frac{H}{2}+\gamma\right]\ t}\,, \tag{22}$$

$$\rho \propto e^{-2[H-\gamma]\ t}\,, \tag{22a}$$

and,

$$\Lambda \propto e^{-H\ t}\,. \tag{22b}$$

It seems reasonable that inflation decreases the energy density and the cosmological term, while N grows exponentially; of course, we take $H > \gamma$.

4. ROTATION OF THE UNIVERSE

A closely related issue is the possibility of a Universal spin. Consider the Newtonian definition of angular momentum L,

$$L = RMv\,, \tag{23}$$

where, R and M stand for the scale-factor and mass of the universe.

For Planck's universe, the obvious dimensional combination of the constants $\bar{h}$, c, and G is,

$$L_{Pl} = \bar{h}\,. \tag{24}$$

From equations (23) and (24), we see that Planck's universe spin has a speed $v = c$. For any other time, we take, then, the spin of the universe as given by

$$L = RMc\,. \tag{25}$$

In the first place, we take the known values of the present universe: $R \approx 10^{28}$ cm, and $M \approx 10^{55}$ g, so that,

$$L = 10^{93}\ \text{cm.g.cm/s} = 10^{120}\bar{h}\,. \tag{26}$$

We have thus, another large number,

$$\frac{L}{\bar{h}} \propto N^{3/2}\,. \tag{27}$$

For instance, for the power law, as in standard cosmology, we would have,

$$L \propto t^{3+9n} = t^{3(1+3n)}\,. \tag{28}$$

For exponential inflation,

$$L \propto e^{\frac{3}{2}[H+2\gamma]\ t}\,. \tag{29}$$

We now may guess a possible angular speed of the universe, on the basis of Dirac's LNH. For Planck's universe, the obvious angular speed would be:

$$\omega_{Pl} = \frac{c}{R_{Pl}} \approx 2\times 10^{43}\ \text{s}^{-1}\,, \tag{30}$$

because Planck's universe is composed of dimensional combinations of the fundamental constants.I recall a paper by Arbab (2004), that attaches a meaning to the above angular speeds, as yielding minimal accelerations in the universe. The argument runs as follows. From manipulation with the constants that represent the universe (c, h, G) we can construct, not only Planck's usual quantities, but also a dimensionally correct acceleration. With this acceleration, we would construct, if we call it a centripetal $a = -\omega^2 R$ term, the angular speed of our present calculation. But Arbab failed to interpret the existing Planck's constant as representing an angular rotation. However, he says that this centripetal acceleration is a consequence of the vacuum energy, and calculates correctly its present value.

In order to get a time-varying function for the angular speed, we recall the Newtonian angular momentum formula,

$$L = R^2 M\omega\,. \tag{31}$$

In the case of a power-law c-variation, we have found, from relation (27), that, $L \propto N^{3/2}$, but we also saw from relation (31) that $L \propto \rho R^5\omega$, because $R = cH^{-1} \propto \sqrt{N}$ and $M \propto \rho R^3 \propto N$.

Then, we find that,

$$\omega = \omega_0 t^{-1+6n} = AR^{-(1-6n)}\,. \qquad (\omega_0, A = \text{constants}) \tag{31a}$$

We are led to admit the following relation:

$$\omega \lessapprox \frac{c}{R}\,. \tag{32}$$

For the present universe, we shall find,

$$\omega \lessapprox 3\times 10^{-18}\ \text{s}^{-1}\,. \tag{33}$$

It can be seen that the present angular speed is too small to be detected by present technology.

For the inflationary model, we carry out a similar procedure:

$$\omega \propto \frac{N^{\frac{3}{2}}}{R^5\rho} = e^{\left[-\frac{9}{2}H+\gamma\right]\ t}\,. \tag{34}$$

The condition for a decreasing angular speed in the inflationary period, is, then,

$$\gamma < \frac{9}{2}H\,. \tag{35}$$

5. PROS AND CONS OF THE PRESENT CALCULATIONS

Critical appraisals of the above calculations center on the four following arguments:

I. Do the time variations, $G(t)$, $\rho(t)$, and $\Lambda(t)$ proposed above violate Einstein's field equations?

II. If Einstein's theory does not apply, which one does? And then, do the new equations reduce to Einstein's in a proper limit?

III. If there is rotation, would it not imply some preferred direction in the universe?

IV. Can order-of-magnitude calculations be valid in order to get insights on the universe?

We now reply:

1. Dirac never proposed LNH as part of GRT (General Relativity Theory), neither do I.

2. Dirac's LNH is a foil for testing hypotheses, like the theoretical frameworks of scalar-tensor cosmologies with lambda (see for instance Berman 2007b). In such theories, our present results may be included (Berman 2007b).

3. According to the Machian approach by Berman (2007c, 2008a), the kind of rotation to be expected in the universe, has no unique axis of rotation; we know that there is a Machian rotation,because each "observer" sees any "observed", far away (at cosmological distances), with the centripetal acceleration that identifies the "Machian rotation". It is a rotation like that of the Gödel universe (Adler, Bazin, & Schiffer 1975).

4. Dirac's universe, though appealing, does not stand as a mathematically correct solution of any gravitational theory, like for instance, general relativity. It is more of a tool that identifies possible physical effects in the universe.

We hope to have clarified the former cons, with the latter pros.

6. CONCLUSIONS

Paraphrasing Dicke (1964a,1964b), the many faces of Dirac's LNH have been shown, as many as there are about Mach's principle. In face of modern Cosmology, the naif theory of Dirac is a foil for theoretical discussion on the foundations of this branch of physical theory. The angular speed found by us matches results by Gödel (see Adler et al. 1975), de Sabbata & Gasperini (1979), and Berman (2007c, 2008b,c).

There is a *no-rotation* condition, for $n = \frac{1}{6}$, in the power-law solution; likewise, with $\gamma = \frac{9}{2}H$, this is the *no-rotation* condition of the inflationary angular speed formula. However, these cases are foreign to the idea of a weak time-varying formula for the fine-structure "constant".

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