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Effects of Public-Security Expenditures on Economic Growth: A stochastic macroeconomic model

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Abstract

This paper examines security-related expenditures aimed at combating organized crime and analyzes their effects on economic growth. For this purpose, a stochastic, macroeconomic, general-equilibrium model is used in order to analyze the accrual of resources appropriated for the fight against organized crime, as well as the effects of exogenous shocks from output and the expenditure of output. This model reveals that a change in the level of economic resources spent by organized crime against the state will elicit a reaction by the government, causing an impact on economic growth due to the elasticity of intertemporal substitution of consumption. Lastly, based on the functional relationships established by the theoretical model and empirical evidence given within a vector autoregression (VAR) model, the relationship between security-related budgetary expenditures and economic growth is discussed.

Key words: general equilibrium, public-security expenditure, economic growth.
JEL **Classification**: C60, C61, D10, E20, E22.

Introduction

Narcotics trafficking and organized crime have existed in Mexico for a long time. Yet in the past six-year presidential term (2006-2012), this issue became a priority in government policies and a series of strategies were adopted to limit the presence of organized crime in certain regions of the country. During the past administration, the police and military sectors were the most dynamic in the realm of security. The government embarked on an assault against organized crime that rhetorically has been compared to war. At the close of the six-year

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period, the government had spent more than 642 billion pesos on its strategy to pacify the country, a figure that more than doubles the 308 billion pesos spent in this area in the previous administration (2000-2006), according to data from the Chamber of Deputies' *Centro de Estudios de las Finanzas Públicas* (CEFP) [Center for Public Finance Studies]. Leaving aside the theoretical polemics regarding whether this constitutes a war or not, the fact is that a large part of the Mexican Army is on the streets, carrying out missions, and fully engaged in fighting organized crime. The problem of narcotics trafficking and organized crime is now comparable in Mexico to that of terrorism in other countries, exemplified by the attacks against the civilian population on September 15, 2008 in Michoacán. This has created concern in the government, which responded by appropriating more funding to confront what is now considered a national-security problem. According to CEFP, during the last year of the 2006-2012 presidential term, budget appropriations for the Secretaries of the Interior (*Secretaría de Gobernación*, Segob), National Defense (*Secretaría de la Defensa Nacional*, Sedena), Navy (*Secretaría de Marina*, Semar), Public Security (*Secretaría de Seguridad Pública*, SSP), and the Attorney General's Office (*Procuraduría General de la República*, PGR), exceeded 153 billion pesos. Table 1 shows the changes in SSP and Sedena expenditures, reflecting the priority given those sectors during the 2006-2012 term, as part of a strategy to fight organized crime. Lastly, federal spending exclusively on the war against narcotics trafficking and organized crime from 2006 to 2011 was approximately 174 billion pesos.

Table **1**
Budget for the 2006-2012 period appropriated for public security and national defense
(millions of current pesos)

Year	*SSP*	*Sedena*
2006	9 274.4	26 031.9
2007	13 664.7	32 200.8
2008	19 711.6	34 861.0
2009	32 916.8	43 623.3
2010	32 437.8	43 632.4
2011	35 519.1	50 039.5
2012	40 536.5	55 611.0

Source: compiled by the authors with data from the Chamber of Deputies' CEFP.

Table 2 shows each secretary's security-related budget, was well as appropriations from the *Fondo de Aportaciones a la Seguridad Pública de los Estados y del Distrito Federal* (FASP) [Fund of Public Security Appropriations for States and the Federal District] for 2011-2012 within the *Programa Nacional de Seguridad Pública* [National Public-Security Program].

Table **2**
Federal budgeted expenditures for national security matters, 2011-2012
(millions of current pesos)

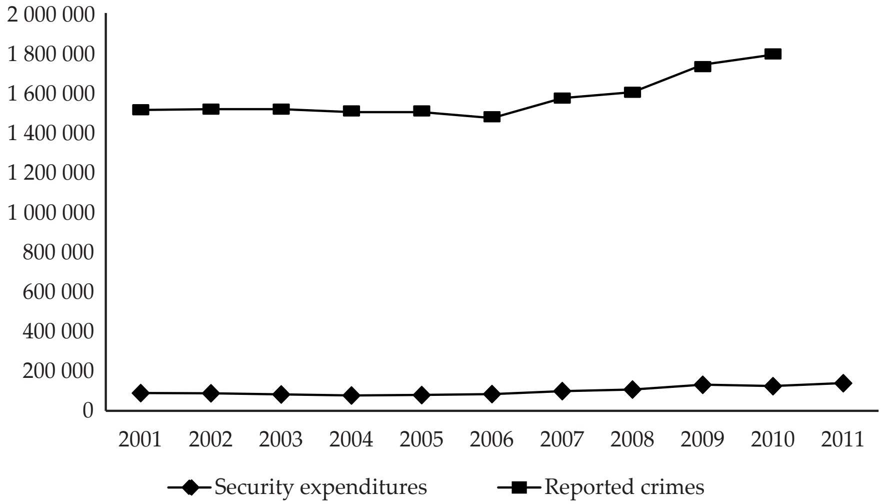
	2012	*2011*
Segob	15 458.2	16 386.1
PGR	14 905.1	11 997.1
SSP	40 536.5	35 519.1
Sedena	55 611	50 039.5
Semar	19 679.7	18 270.2
FASP	7 373.7	7 124.4
Total	153 564.2	139 336.4

Source: compiled by the authors with data from the Chamber of Deputies' CEFP and from the *Instituto Nacional de Estadística y Geografía* (INEGI).

The significant increase in security-related expenditures in participating governmental offices has led to an increasing tendency to "report" crimes, as shown in Graph 1. Further, this Graph clearly shows how security expenditures have evolved over the past ten years, from 90 billion pesos in 2001 to 139 billion in 2011.

Clearly, the increase in security-related expenditures has been used broadly to address the threat from organized crime. In principle, we might be led to think that by channeling more resources to security, other output sectors would be affected, thus leading to dampened economic growth over the long term, following Yang, Lin, and Chen (2012). In an empirical regression study of 71 countries from 1969 to 1989, Landau (1993) shows the existence of a non-linear relationship between military expenditure and economic growth. On the other hand, studies by Yang, Lin, and Chen (2012) find a negative relationship between antiterrorist expenditures and social welfare. Further, research by Lin and Lee (2012) suggests that growth is a function of the degree of actors' risk aversion. Finally, Deger and Sen (1983) show that defense expenditures could stimulate demand and, thus, increased employment and output, which would exert a positive effect on economic development.

Graph 1
Changes in security expenditures (millions of 2011 pesos) and reported crimes (number of crimes)



Source: compiled by the authors with data from the Chamber of Deputies' CEFP.

A current topic is the impact of increased security-related government spending on different macroeconomic variables such as investment or GDP growth. This investigation develops a model within the framework of intertemporal optimization that endeavors to explain the effects on consumption, investment, and economic growth of increases in security-related expenditures directed against organized crime. The model deals with this expenditure first as a consumption good and then as an investment good. Similarly, Gong and Zou's work (2003), developed within neoclassical theory, is our point of departure for analyzing the impact of security spending on economic growth from a perspective of stochastic optimization.

This theoretical model is proposed within a stochastic environment, *i.e.*, although a budget expense might be appropriated to fight organized crime, disbursements can change continuously in response to circumstances on the ground. The problem can also be analyzed as a two-stage game in which the response, of both the government and criminal groups, is a function of the amount of funds appropriated for fighting crime. Although public-security expenditures and military disbursements are not one and the same, a ratio from both can be calculated to represent the expenditure channeled to fight organized crime; that ratio should be taken as the security expenditure in the model proposed herein. To resolve the model, we begin with assumptions from neoclassical orthodoxy of individuals who maximize their satisfaction, *i.e.*, an AK-styled production function. Thus, macroeconomic results will have microeconomic foundations. Further, we consider a production function, such as the one in Turnovsky (2000), and a strictly increasing concave utility function. Within the theoretical framework explained herein, we shall examine the impact on growth of marginal changes of the amount of resources channeled to fight organized crime. A relevant result from the model indicates that growth is a function of an individual's elasticity of intertemporal substitution.

In the empirical section, we posit an autoregressive vectors (VAR) model, based on functional relationships that originate in the theoretical model, to explore the relationship between security-related budgetary expenditures and gross domestic product (GDP). Although the security budget and GDP have similar growth paths, from 2000 to 2012 these variables do not have a causal relationship in both directions in the sense of Granger.

This paper is organized in four sections: the first is an introduction; the following section covers security expenditures as a consumption good; next, security expenditures are seen as an investment, thus allowing us to analyze the dynamic of weapons and equipment accumulation aimed at fighting organized crime. In sections two and three, a solution is obtained regarding forecasts of economic growth. The fourth section takes up empirical evidence regarding the relationship between security-related budgetary expenditures and growth, and the final section discusses our conclusions.

Security expenditures as a consumption good

This model is set in a context in which the Mexican state is in a military confrontation with organized crime. We assume that preferences are defined by consumption (c_t), the level of security expenditures appropriated for fighting organized crime (s_t), and the level of expenditure by organized crime in its fight against the state (s_t^*), so that utility is determined as a function of these three variables, *i.e.*, $u(c_t, s_t, s_t^*)$. Although the expenditure of organized crime is included in the utility function, this does mean that this economic actor necessarily makes decisions regarding the function, nor that it has a positive relationship with utility; in fact, this relationship is negative. In this regard, we posit a utility function that generates no satisfaction for organized crime should it increase its expenditure, and meets all theoretical characteristics of neoclassical orthodoxy. We posit a production function with AK-styled stochastic technology, as in Merton (1975) and Turnovsky (2000), given by:

$$\mathrm{d}Y_t = [Ak_t \mathrm{d}t + Ak_t\, \sigma_y \mathrm{d}W_t] \tag{1}$$

where Y_t is output, A is a positive constant, k_t is the stock of capital that, when multiplied by $\mathrm{d}t$, defines the determinist component per unit of time, σ_y is the volatility parameter of GDP, and $\mathrm{d}W_t$. is the stochastic element that expresses the impacts not forecast in GDP. Further, we assume that organized crime's expenditure in its war against the state follows a stochastic process expressed as:

$$\frac{\mathrm{d}s_t^*}{s_t^*} = (\alpha \mathrm{d}t + \sigma \mathrm{d}V_t) \tag{2}$$

where α is the average level of expenditure (tendency) by organized crime in its war against the state, σ is the volatility of the expenditure, and $\mathrm{d}V_t$ is the normally-distributed stochastic component with zero mean and $\mathrm{d}t$ variance. We also assume that $\mathrm{Cov}(\mathrm{d}W_t, \mathrm{d}V_t) = \sigma_{WV}\mathrm{d}t$, where $\sigma_{WV}\mathrm{d}t > 0$, indicating that the correlation between the shocks of expenditure, undertaken both by organized crime and the state is positive, thus expressing the arms build- up among them.

Once security-related expenditures are included in consumption goods, the marginal increase in the domestic-income identity[1] can be expressed as $\mathrm{d}Y_t = [c_t\mathrm{d}t + s_t\mathrm{d}t + s_t^*\mathrm{d}t] + \mathrm{d}k_t$, where $\mathrm{d}k$ is the marginal change in capital stock. The budget constraint can be expressed by $\mathrm{d}k_t = \mathrm{d}Y_t - c_t\mathrm{d}t - g_T\mathrm{d}t - s_t^*\mathrm{d}t$, where total governmental expenditure is $G_t\mathrm{d}t = \bar{g}\mathrm{d}t + s_t\mathrm{d}t$, which includes current expenditure ($\bar{g}$) and security-related expenditures s_t. By virtue of equations [1] and [2], the marginal increase of capital is obtained:

$$\mathrm{d}k_t = \left[Ak_t - c_t - G_t - s_t - s_t^*\right]\mathrm{d}t + Ak_t\sigma_y \mathrm{d}W_t$$

[1] For the sake of simplicity, the economy is assumed to be closed.

If current expenditure is considered to be zero, *i.e.*, when $\bar{g}\,\mathrm{d}t = 0$, in order to avoid unnecessary calculations for the matter at hand, we have then:

$$\mathrm{d}k_t = \left[Ak_t - c_t - s_t - s_t^*\right]\mathrm{d}t + Ak_t\sigma_y \mathrm{d}W_t$$

Optimization problem (model 1)

We begin with the assumption that individuals in this economy seek to maximize their utility and will thus have to choose optimum security-related consumption and expenditure trajectories. The selection problem is subordinate to the dynamic of capital and to organized crime's expenditure combatting the state, discounted with a subjective rate, $0 < \beta < 1$, within the framework of [2] and [1], in other words:

$$\text{Maximize } \mathrm{E}_0\left[\int_0^\infty u\left(c_t, s_t, s_t^*\right)e^{-\beta t}\mathrm{d}t\right]$$

$$\text{s.t. } \mathrm{d}k_t = \left[Ak_t - c_t - s_t - s_t^*\right]\mathrm{d}t + Ak_t\sigma_y \mathrm{d}W_t$$

$$\mathrm{d}s_t^* = s_t^*\left[\alpha \mathrm{d}t + \sigma \mathrm{d}V_t\right]$$

This maximization problem can be studied from a stochastic optimal control theory.[2] Defining:

$$J\left(k_t, s_t^*, t\right) = \max \mathrm{E}_0\left[\int_0^\infty u\left(c_t, s_t, s_t^*\right)e^{-\beta t}\mathrm{d}t\right]$$

then:

$$\begin{aligned} J\left(k_t, s_t^*, t\right) &= \max \mathrm{E}_0\left[\int_t^{t+\mathrm{d}t} u\left(c_t, s_t, s_t^*\right)e^{-\beta t}\mathrm{d}t + \int_{t+\mathrm{d}t}^{\infty}\left(c_t, s_t, s_t^*\right)e^{-\beta t}\mathrm{d}t\right] \\ &= \max \mathrm{E}_0\left[\int_t^{t+\mathrm{d}t} u\left(c_t, s_t, s_t^*\right)e^{-\beta t}\mathrm{d}t + J\left(k_t + \mathrm{dk}_t, s_t^* + \mathrm{d}s_t^*, t + \mathrm{d}t\right)\right] \end{aligned}$$

[2] For a quick and easy explanation of this methodology, see Venegas-Martínez (2008).

If we apply to the previous equation both the mean value theorem $\int_a^b f(x)\mathrm{d}x = f(a)(b-a) + o(b-a)$, and the Taylor expansion, we obtain:

$$J\left(k_t, s_t^*, t\right) = \max \mathrm{E}_0\left[u\left(c_t, s_t, s_t^*\right)e^{-\beta t}\mathrm{d}t + o(\mathrm{d}t) + J\left(k_t, s_t^*, t\right) + \mathrm{d}\,J\left(k_t, s_t^*, t\right)\right]$$

equivalent to:

$$0 = \max \mathrm{E}_0\left[u\left(c_t, s_t, s_t^*\right)e^{-\beta t}\mathrm{d}t + o(\mathrm{d}t) + \mathrm{d}J\left(k_t, s_t^*, t\right)\right]$$

The stochastic differential $\mathrm{d}J(k,s^*,t)$ is calculated with Itō's lemma:

$$\left[J_t + J_k(F(k_t) - c_t - s_t - s_t^*) + \frac{1}{2}J_{kk}F(k)^2\sigma_y^2 + J_{s^*}\alpha s_t^* + \frac{1}{2}J_{s^*s^*}\sigma^* s_t^{*2} + J_{ks^*}F(k_t)s_t^*\sigma_{WV}\right]\mathrm{d}t$$
$$+J_k F\left(k_t\right)\mathrm{d}W_t + \sigma s_t^* J_{s^*}\mathrm{d}V_t$$

By taking the expected value, the stochastic terms $\mathrm{d}W_t$ and $\mathrm{d}V_t$ become zero. We then divide by $\mathrm{d}t$ and take the limit when $\mathrm{o}(\mathrm{d}t)/\ \mathrm{d}t \to 0$, leading to the following expression:

$$0 = \max \mathrm{E}_0\left[u\left(c_t, s_t, s_t^*\right)e^{-\beta t} + J_t + J_k(F(k_t) - c_t - s_t - s_t^*) + \frac{1}{2}J_{kk}F(k)^2\sigma_y^2 + J_{s^*}\alpha s_t^*\right.$$
$$\left. + \frac{1}{2}J_{s^*s^*}\sigma^* s_t^{*2} + J_{ks^*}F(k_t)s_t^*\sigma_{WV}\right] \qquad [3]$$

where $F(k_t) = Ak_t$. This is known as the Hamilton, Jacobi, and Bellman (H-J-B) equation.

First-order conditions

The value function is defined by $J(k_t, s_t^*, t) = V(k_t, s_t^*)e^{-\beta t}$, and the partial derivatives with respect to the state and control variables are taken, *i.e.*:

$$\frac{\partial J}{\partial t}=-\beta V(k_t,s_t^*)e^{-\beta t},\frac{\partial J}{\partial k_t}=V_k(k_t,s_t^*)e^{-\beta t},\frac{\partial J}{\partial s_t^*}=V_{s^*}(k_t,s_t^*)e^{-\beta t}$$

$$\frac{\partial^2 J}{\partial k_t^2}=V_{kk}(k_t,s_t^*)e^{-\beta t},\frac{\partial^2 J}{\partial s_t^{*2}}=V_{s^*s^*}(k_t,s_t^*)e^{-\beta t},\frac{\partial J}{\partial k_t s_t^*}=V_{ks^*}(k_t,s_t^*)e^{-\beta t}$$

By substituting the partial derivatives from the value function in equation [3], we have:

$$\begin{aligned}0=\max \mathrm{E}_0\Big[&u\left(c_t,s_t,s_t^*\right)e^{-\beta t}-\beta V(k_t,s_t^*)e^{-\beta t}+V_k(k_t,s_t^*)e^{-\beta t}(F(k_t)-c_t-s_t-s_t^*)\\&+\frac{1}{2}V_{kk}(k_t,s_t^*)e^{-\beta t}F\left(k_t\right)^2\sigma_y^2+V_{s^*}(k_t,s_t^*)e^{-\beta t}\alpha s_t^*+\frac{1}{2}V_{s^*s^*}(k_t,s_t^*)e^{-\beta t}\sigma^2 s_t^*\\&+V_{ks^*}(k_t,s_t^*)e^{-\beta t}F\left(k_t\right)s_t^*\sigma_{WV}\Big]\end{aligned} \quad [4]$$

As a solution, we propose the $V(k_t,s_t^*)=\rho k^{1-\delta}(s_t^*)^{-\varphi}$ function, and calculate the partial derivatives with respect to the state variables. In this case, φ is the elasticity of substitution of the expenditure by organized crime and ρ is a value function parameter. Thus:

$$V(k_t,s_t^*)=\rho k_t^{1-\delta}\left(s_t^*\right)^{-\varphi},V_k(k_t,s_t^*)=\rho\left(1-\delta\right)k_t^{-\delta}\left(s_t^*\right)^{-\varphi}$$

$$V_{kk}(k_t,s_t^*)=-\rho\delta\left(1-\delta\right)k_t^{-\delta-1}\left(s_t^*\right)^{-\varphi},V_{ks^*}(k_t,s_t^*)=-\rho\varphi\left(1-\delta\right)k_t^{-\delta}\left(s_t^*\right)^{-(\varphi+1)}$$

$$V_{s^*}(k_t,s_t^*)=-\rho k_t^{1-\delta}\varphi\left(s_t^*\right)^{-(\varphi+1)}$$

By substituting the partial derivatives of the $V(k_t, s_t^*)$ function from [4] we have:

$$\begin{aligned}0=\Big[&u\left(c_t,s_t,s_t^*\right)e^{-\beta t}-\beta\rho k_t^{1-\delta}\left(s_t^*\right)^{-\varphi}+\rho\left(1-\delta\right)k_t^{-\delta}\left(s_t^*\right)^{-\varphi}(F(k_t)-c_t-s_t-s_t^*)\\&-\frac{1}{2}\rho\delta\left(1-\delta\right)k_t^{-\delta-1}\left(s_t^*\right)^{-\varphi}F\left(k_t\right)^2\sigma_y^2-\rho k_t^{1-\delta}\varphi\left(s_t^*\right)^{-(\varphi+1)}\alpha s_t^*\\&+\frac{1}{2}\rho\varphi\left(\varphi+1\right)k_t^{1-\delta}\left(s_t^*\right)^{-(\varphi+2)}\sigma^2 s_t^*-\rho\varphi\left(1-\delta\right)k_t^{-\delta}\left(s_t^*\right)^{-(\varphi+1)}F\left(k_t\right)s_t^*\sigma_{WV}\Big]\end{aligned} \quad [5]$$

In what follows we posit a suitable functional form for utility in order to find a specific solution.

Utility function and optimal decisions

A Cobb-Douglas-style utility function is proposed with two goods that generate satisfaction, *i.e.*, consumption and expenditure geared to fighting organized crime:

$$\bar{U}(c_t, s_t) = c_t^{\theta} s_t^{1-\theta}$$

If we assume that another factor causes no utility, *i.e.*, organized crime's expenditure in fighting the state, we then have another utility function such that:

$$\widehat{U}(c_t, s_t, s_t^*) = (c_t^{\theta} s_t^{1-\theta})^{1-\delta} \left(s_t^{*-\lambda}\right)^{\delta}$$

and by redefining $\lambda\delta = \varphi$, given that $(1/1-\delta)$ is constant, when multiplied by the utility function preferences are not changed, and thus the utility function can be defined as:

$$U(c_t, s_t, s_t^*) = c_t^{\theta(1-\delta)} \left(1/1-\delta\right)\left[s_t^{(1-\theta)(1-\delta)} \left(s_t^*\right)^{-\varphi}\right]$$

The parameters satisfy $0 < \theta < 1$; $\varphi > 0$ when $0 < \delta < 1$ and $\varphi < 0$ when $\delta > 1$; restrictions on φ and δ are to guarantee that an increase in expenditure by organized crime in its fight against the state will not, under any circumstances, generate utility, in other words $\partial u_t/\partial s_t^* < 0$, for all possible scenarios. From [5], and incorporating the utility function previously proposed, we have:

$$\begin{aligned}\Big[& c_t^{\theta(1-\delta)} \left(1/1-\delta\right) s_t^{(1-\theta)(1-\delta)} \left(s_t^*\right)^{-\varphi} - \beta\rho k_t^{1-\delta} \left(s_t^*\right)^{-\varphi} + \rho\left(1-\delta\right) k_t^{-\delta} \left(s_t^*\right)^{-\varphi} \left(F(k_t) - c_t - s_t - s_t^*\right) \\ & - \frac{1}{2}\rho\delta\left(1-\delta\right) k_t^{-\delta-1} \left(s_t^*\right)^{-\varphi} F\left(k\right)^2 \sigma_y^2 - \rho k_t^{1-\delta} \varphi \left(s_t^*\right)^{-(\varphi+1)} \alpha s_t^* \\ & + \frac{1}{2}\rho\varphi\left(\varphi+1\right) k_t^{1-\delta} \left(s_t^*\right)^{-(\varphi+2)} \sigma^2 s_t^* - \rho\varphi\left(1-\delta\right) k_t^{-\delta} \left(s_t^*\right)^{-(\varphi+1)} F\left(k_t\right) s_t^* \sigma_{WV} \Big] = 0\end{aligned} \tag{6}$$

By applying the first-order conditions regarding consumption and the level of security-related expenditure in equation [6], the following expressions are obtained:

$$\begin{aligned} \theta\left(c_t^{\theta}s_t^{1-\theta}\right)^{-\delta}\left(s_t^{*}\right)^{-\delta}c_t^{\theta-1}s_t^{1-\theta} &= \rho(1-\delta)k_t^{-\delta}\left(s_t^{*}\right)^{-\varphi} \\ (1-\theta)\left(c_t^{\theta}s_t^{1-\theta}\right)^{-\delta}\left(s_t^{*}\right)^{-\varphi}c_t^{\theta}s_t^{-\theta} &= \rho(1-\delta)k_t^{-\delta}\left(s_t^{*}\right)^{-\varphi} \end{aligned} \qquad [7]$$

If we define total consumption (C_t) as the sum of individual consumption and the level of expenditure for fighting organized crime, then $C_t = c_t + s_t$, $c_t = \theta C_t$, $s_t = (1-\theta)C_t$, $0 < \theta < 1$. If C_t is substituted in [7], we obtain a consumption/capital ratio that will be useful later on in evaluating GDP growth:

$$\frac{C_t}{k_t} = \left[\rho(1-\delta)\left[\theta^{\theta}(1-\theta)^{1-\theta}\right]^{\delta-1}\right]^{-\frac{1}{\delta}} \qquad [8]$$

Thus, by substituting [8] in [6], we obtain an expression that leads to the consumption/capital ratio:

$$\begin{aligned} 0 = \Bigg[&\left(\frac{1}{1-\delta}\right)\left[\theta^{\theta}(1-\theta)^{1-\theta}\right]^{\delta-1}\left(s_t^{*}\right)^{-\varphi}k_t^{1-\delta}\left[\rho(1-\delta)\left[\theta^{\theta}(1-\theta)^{1-\theta}\right]^{\delta-1}\right]^{\frac{\delta-1}{\delta}} \\ &-\beta\rho k_t^{1-\delta}\left(s_t^{*}\right)^{-\varphi}+\rho(1-\delta)k_t^{-\delta}\left(s_t^{*}\right)^{-\varphi}(Ak_t-C_t-s_t^{*})-\rho k_t^{1-\delta}\varphi\left(s_t^{*}\right)^{-(\varphi+1)}\alpha s_t^{*} \\ &-\rho\varphi(1-\delta)k_t^{-\delta}\left(s_t^{*}\right)^{-\varphi}A\sigma_{WV}+\frac{1}{2}\rho(1-\delta)(-\delta)k_t^{-(\delta+1)}\left(s_t^{*}\right)^{-\varphi}\sigma_y^2A^2k_t^2 \\ &+\frac{1}{2}\rho\varphi(\varphi+1)k_t^{1-\delta}\left(s_t^{*}\right)^{-\varphi-2}\sigma^2s_t^{*2}\Bigg] \end{aligned} \qquad [9]$$

After dividing equation [9] by $(s^{*})^{-\varphi}$, factoring the term $[\cdot]^{(\delta-1)/\delta}$ and dividing by $k^{1-\delta}$ and ρ, the consumption/capital ratio can be expressed by:

$$\frac{C_t}{k_t} = \frac{\varphi\alpha+\frac{1}{2}(1-\delta)\delta A^2\sigma_y^2-\frac{1}{2}\varphi(\varphi+1)\sigma^2-(1-\delta)A+\beta+(1-\delta)A\varphi\sigma_{WV}+\frac{s_t^{*}}{k_t}(1-\delta)}{\delta}$$

and given that $c_t + s_t = C_t$ and $F(k_t) = Ak_t$, the dynamic equation of capital stock is given by:

$$\begin{aligned}
\mathrm{d}k_t &= \left[F(k_t) - c_t - s_t - s_t^*\right]\mathrm{d}t + F(k_t)\sigma_y \mathrm{d}W_t \\
&= [Ak_t - C_t - s_t^*]\mathrm{d}t + Ak_t\sigma_y \mathrm{d}W_t \\
&= k_t\left[\left(A - \frac{C_t}{k_t} - \frac{s_t^*}{k_t}\right)\mathrm{d}t + A\sigma_y \mathrm{d}W_t\right]
\end{aligned}$$

Therefore expectations regarding the consumption growth rate and the capital stock, denoted by ω_1, satisfy:

$$\omega_t = E\frac{\frac{\mathrm{d}c_t}{\mathrm{d}t}}{c_t} = E\frac{\frac{\mathrm{d}k_t}{\mathrm{d}t}}{k_t} = \frac{\frac{k_t\left[\left(A - \frac{C_t}{k_t} - \frac{s_t^*}{k_t}\right)\mathrm{d}t + A\sigma_y \mathrm{d}W_t\right]}{\mathrm{d}t}}{k_t} = \left(A - \frac{C_t}{k_t} - \frac{s_t^*}{k_t}\right)$$

Given the above, we arrive at a differential equation that leads to the capital stock, $\mathrm{d}k_t = k_t[\varphi_1 \mathrm{d}t + A\sigma_y \mathrm{d}W_t]$, whose solution, with an initial situation $k(0) = k_0$, is given by:

$$k_t = k_0 e^{\left(\omega_1 - \frac{1}{2}A^2\sigma_y^2\right)t + \sigma_y A\,W_t}$$

The stochastic expenditure trajectory of organized crime is given by the following expression:

$$s_t^* = s_0^* e^{\left(\alpha - \frac{1}{2}\sigma^2\right)t + \sigma V_t}$$

The conditions that guarantee a positive consumption/capital ratio are given by:

$$\lim_{x\to\infty} \mathrm{E}\left[\delta k_t^{1-\delta}\left(s_t^*\right)^{-\varphi} e^{-\beta t}\right] = 0$$

$$(1-\delta)\left[A - \frac{C_t}{k_t} - \frac{1}{2}\delta A^2\sigma_y^2 - \varphi A\sigma_{WV} - \frac{s_t^*}{k_t}\right] - \varphi\left[\alpha - \frac{1}{2}(\varphi+1)\sigma^2\right] - \beta < 0$$

Theoretical results and analysis

This analysis focuses on the effects of expenditure on growth and capital stock. With regards to average expenditure that organized crime assigns to fighting the state, growth in the average tendency of the threat posed by crime organizations affects economic growth as follows:

$$\frac{\partial \omega_1}{\partial \alpha} = \frac{-\varphi}{\delta}$$

Thus, $\varphi > 0$ when $0 < \delta < 1$, and $\varphi < 0$ when $\delta > 1$. If $\delta > 1$, $\partial\omega 1/\partial\alpha > 0$, and if $0 < \delta < 1$, $\partial\omega_1/\partial\alpha < 0$. This means that high or low growth in the average tendency of organized crime's expenditure to fight the government entails economic growth or contraction, if and only if the elasticity of intertemporal substitution of consumption, given by $1/\delta$, is relatively small or large. When organized crime increases the average level of funds to fight the government, a reaction on the part of the state ensues: the government will also increase it expenditure to counter the criminal element. Given that the state's security-related expenditures has a direct relationship with the utility function, then the marginal utility of the state will increase security-related expenditures. The fact that the government increases its expenditure to fight organized crime leads to a reduction in capital investment when the elasticity of intertemporal substitution of consumption is relatively elastic, in other words when $0 < \delta < 1$. Therefore, in the long run, the rate of growth will decrease. Further, when $\delta > 1$, this actor's consumption elasticity of substitution is small, thus reducing consumption, increasing investment, and producing greater income.

In terms of the stochastic impact of the threat of organized crime on economic growth, this is given by:

$$\frac{\partial \omega_1}{\partial \sigma^2} = \frac{1}{2} \cdot \frac{\varphi}{\delta}(\varphi + 1)$$

Thus, we have that $\partial\omega_1/\partial\sigma^2 > 0$ when $0 < \delta < 1$ or when $\varphi < -1$ and $\delta > 1$, and $\partial\omega_1/\partial\sigma^2 < 0$ when $\delta > 1$ and $-1 < \varphi < 0$. These results suggest that a high elasticity of intertemporal substitution of consumption of individuals (excluding criminals) who participate in the economy will lead to economic growth when there is greater volatility in the threat posed by organized crime. Yet we should exercise caution, since the threat of organized crime may lead to greater negative utility, *i.e.*, that it will entail a large absolute value of φ.

The stochastic impacts (due to the volatility of output) in the marginal increase of GDP have the following effect on economic growth:

$$\frac{\partial \omega_1}{\partial \sigma_y^2} = -\frac{1}{2} A(1-\delta)$$

So, $\partial\omega_1 / \partial\sigma_y^2 > 0$ when $\gamma > 1$, and $\partial\omega_1 / \partial\sigma_y^2 < 0$ when $0 < \delta <$. From this we conclude that a stochastic impact in the marginal change of GDP entails economic growth when the elasticity of intertemporal substitution of consumption of individuals (excluding criminals) is relatively small.

Expenditure in security matters as an investment good (model 2)

In this section security-related investment is included as an investment good, meaning that capital accumulation can be seen as an arms race (ability of public security forces to fight organized crime). If the state's weapons stock is denoted by s_t and organized crime's weapon stock is denoted by s_t^*, the state's total wealth (r_t) will be the sum of the stock of capital and weapons, in other words, $r_t = k_t + s_t$; while the total wealth of narcotics traffickers and organized crime (r_t^*) will be the sum of its stock of capital and weapons, *i.e.*, $r_t^* = k_t^* + s_t^*$.

The utility of persons participating in the economy is defined by consumption c_t, total wealth r_t, and the total wealth of organized crime r_t^*, thus obtaining $u(c_t, r_t, r_t^*)$. Both levels of wealth are included in the utility function because they represent a realistic view of the strength of a government and its status in the absolute monopoly of force. As in the previous section, the fact that the wealth of organized crime is included in the utility function does not imply that the actor makes decisions regarding this variable. In fact, the wealth of organized crime is included in the utility function to indicate that an increase in this variable generates negative utility. Further, a high capital stock always leads to a larger GDP, contributing to a higher expenditure in security and a greater accumulation of weapons, but there is accumulation without a substantial capital reserve and so GDP growth is unsustainable over the long run.[3]

[3] Several authors have included an individual's wealth in the utility function for varying purposes, for example, Gong and Zou (2003).

If we assume a utility function $u(c_t, r_t, r_t^*)$ and also posit that the wealth of organized crime follows a stochastic tendency such that:

$$\frac{\mathrm{d}r_t^*}{r_t^*} = \alpha_{r^*}\mathrm{d}t + \sigma_{r^*}\mathrm{dV}_t,\ \mathrm{d}V_t \sim N(0, \mathrm{d}t)$$

where α_{r*} is the average trend of the wealth of organized crime, σ_{r*} is the volatility of this wealth, and $\mathrm{d}V_t$ stands for unexpected changes. Note that the marginal change in wealth can be expressed in terms of the marginal changes of capital and the stock of weapons (or other resources used to confront organized crime), which can be represented as:

$$\begin{gathered} \mathrm{d}r_t = \mathrm{d}k_t + \mathrm{d}s_t \\ \mathrm{d}r_t = \mathrm{d}Y_t - c_t\mathrm{dt} - s_t^*\mathrm{dt} \end{gathered} \qquad [10]$$

Equation [10] shows that the marginal increase in wealth (capital and weapons) is equal to the marginal increase in savings (output less consumption and the paramilitary expenditure of organized crime). Individuals now will choose their stock of capital, weapons, and consumption to maximize their utility, with a subjective discount rate.

As is the case in the first model, we assume that the GDP has a stochastic behavior as determined by:

$$\mathrm{d}Y_t = Ak_t + Ak\sigma_{ty}\mathrm{d}W_t,\ \mathrm{d}W_t \sim N(0,\mathrm{d}t) \qquad [11]$$

If, as before, we denote $F(k_t) = Ak_t$ and we set the stock of state weapons as $q = s_t/(k_t + s_t)$ or $(1 - q) = k_t/(k_t + s_t)$, and by virtue of [11] and [10], it follows that:

$$\mathrm{d}r_t = \left[A\left((1-q)r_t\right) - c_t - s_t^*\right]\mathrm{d}t + A\left[(1-q)r_t\right]\sigma_y\mathrm{d}W_t$$

$$F[(1 - q)r_t] = A[(1 - q)r_t]$$

Thus:

$$\mathrm{d}r_t = \left[F\left((1-q)r_t\right) - c_t - s_t^*\right]\mathrm{d}t + F\left[(1-q)r_t\right]\sigma_y\mathrm{d}W_t$$

Once this restriction is set, individuals can optimize their utility.

Decision problem (model 2)

The problem for individuals lies in maximizing their utility, with a subjective discount rate β, so that:

$$\text{Maximize } \mathrm{E}_0\left[\int_0^{\infty} u\left(c_t, r_t, r_t^*\right)e^{-\beta t}\mathrm{d}t\right]$$

$$\text{s.t. } \mathrm{d}r_t = \left[F\left((1-q)r_t\right) - c_t - s_t^*\right]\mathrm{d}t + F\left[(1-q)r_t\right]\sigma_y \mathrm{d}W_t$$

$$\mathrm{d}r_t^* = \alpha_{r^*} r_t^* \mathrm{d}t + \sigma_{r^*} r_t^* \mathrm{d}V_t$$

This problem of optimal stochastic control is resolved in a manner similar to the first model, and we thus obtain the following equation:

$$0 = \max \mathrm{E}_t\left[u\left(c_t, r_t, r_t^*\right)e^{-\beta t}\mathrm{d}t + o(\mathrm{d}t) + \mathrm{d}\,J\left(r_t, r_t^*, t\right)\right]$$

Once Itō's lemma is applied, we have:

$$\left[J_t + J_r\left[F\left((1-q)r_t\right) - c_t - s_t^*\right] + \frac{1}{2}J_{rr}F\left[(1-q)r_t\right]^2\sigma_y^2 + J_{r^*}\alpha_{r^*}r^* + \frac{1}{2}J_{r^*r^*}\sigma_{r^*}^2 r^{*2}\right.$$
$$\left. + F\left[(1-q)r_t\right]r_t^* J_{rr^*}\sigma_{WV}\right]\mathrm{d}t + F\left[(1-q)r_t\right]J_r\sigma_y \mathrm{d}W_t + \sigma_{r^*}r_t^* J_{r^*}\mathrm{d}V_t$$

The previous expression is divided by $\mathrm{d}t$ and then the limit is taken when $o(\mathrm{d}t)/\mathrm{d}t \to 0$, which leads us to the expression known as the H-J-B equation:

$$0 = \max\left[u\left(c_t, r_t, r_t^*\right)e^{-\beta t}\mathrm{d}t + J_t + J_r\left[F\left((1-q)r_t\right) - c_t - s_t^*\right] + \frac{1}{2}J_{rr}F\left[(1-q)r_t\right]^2\sigma_y^2\right.$$
$$\left. + J_{r^*}\alpha_{r^*}r_t^* + \frac{1}{2}J_{r^*r^*}\sigma_{r^*}^2 r^{*2} + F\left[(1-q)r_t\right]r_t^* J_{rr^*}\sigma_{WV}\right]$$

By assuming a value function such as: $J\left(r_t, r_t^*, t\right) = V\left(r_t, r_t^*\right)e^{-\beta t}$, we then have:

$$0=\max\Big[u\left(c_t,r_t,r_t^*\right)e^{-\beta t}+V_t\left(r_t,r_t^*\right)e^{-\beta t}+V_r\left[F\left(\left(1-q\right)r_t\right)-c_t-s_t^*\right]e^{-\beta t}$$
$$+\frac{1}{2}V_{rr}F\left[\left(1-q\right)r_t\right]^2\sigma_y^2e^{-\beta t}+V_{r^*}\alpha_{r^*}r_t^{\,*}e^{-\beta t}+\frac{1}{2}V_{r^*r^*}\sigma_{r^*}^2r_t^{*2}e^{-\beta t}$$
$$+F\left[\left(1-q\right)r_t\right]r_t^*V_{rr^*}\sigma_{WV}e^{-\beta t}\Big]$$

which takes us to:

$$0=\max\Big[u\left(c_t,r_t,r_t^*\right)-\beta V+V_r\left[F\left(\left(1-q\right)r_t\right)-c_t-s_t^*\right]+\frac{1}{2}V_{rr}F\left[\left(1-q\right)r_t\right]^2\sigma_y^2$$
$$+V_{r^*}\alpha_{r^*}r_t^{\,*}+\frac{1}{2}V_{r^*r^*}\sigma_{r^*}^2r_t^{*2}+F\left[\left(1-q\right)r_t\right]r_t^*V_{rr^*}\sigma_{WV}\Big] \qquad [12]$$

Solution and growth rate of wealth

We posit $V\left(r_t,r_t^*\right)=Xr^{1-\varepsilon-\lambda}\left(r_t^*\right)^{\lambda}$ as a possible solution, and we assume that the utility function takes the following form: $u\left(c_t,r_t,r_t^*\right)=c_t^{1-\varepsilon}/\left(1-\varepsilon\right)\left(r_t/r_t^*\right)^{-\lambda}$, thus $0<\varepsilon<1$ when $-1<\lambda<0$, and $\varepsilon>1$ when $\lambda>1$, which guarantees that an increase in the wealth of organized crime will generate negative utility.

Let c_t, r_t, r_t^* be optima in light of equation [12] and, accounting for the utility function, we obtain:

$$0=\max\left[\frac{c_t^{1-\varepsilon}}{1-\varepsilon}\left(\frac{r_t}{r_t^*}\right)^{-\lambda}-\beta Xr_t^{1-\varepsilon-\lambda}\left(r_t^*\right)^{-\lambda}+\left(1-\varepsilon-\lambda\right)X\left(r_t^*\right)^{\lambda}r_t^{-\varepsilon-\lambda}\left[F\left(\left(1-q\right)r_t\right)-c_t-s_t^*\right]\right.$$
$$+\frac{1}{2}\left(1-\varepsilon-\lambda\right)\left(-\varepsilon-\lambda\right)X\left(r_t^*\right)^{\lambda}r_t^{-\varepsilon-\lambda-1}F\left[\left(1-q\right)r_t\right]^2\sigma_y^2+\lambda Xr_t^{1-\varepsilon-\lambda}\left(r_t^*\right)^{\lambda}\alpha_{r^*}r_t^* \qquad [13]$$
$$\left.+\frac{1}{2}\lambda\left(\lambda-1\right)Xr_t^{1-\varepsilon-\lambda}\left(r_t^*\right)^{\lambda-2}\sigma_{r^*}^2r_t^{*2}+\lambda\left(1-\varepsilon-\lambda\right)X\left(r_t^*\right)^{\lambda-1}r_t^{-\varepsilon-\lambda}r_t^*F\left[\left(1-q\right)r_t\right]\sigma_{WV}\right]$$

If equation [13] is derived with respect to consumption:

$$c^{-\varepsilon}\left(\frac{r_t}{r_t^*}\right)^{-\lambda}=\left(1-\varepsilon-\lambda\right)X\left(r_t^*\right)^{\lambda}r_t^{-\varepsilon-\lambda}$$

In order to obtain the consumption/wealth ratio, an important factor that explains growth, we have then:

$$c^{-\varepsilon} = (1-\varepsilon-\lambda) X r_t^{-\varepsilon}$$

$$\frac{c_t^{-\varepsilon}}{r_t^{-\varepsilon}} = (1-\varepsilon-\lambda) X$$

We thus obtain the equation sought, expressed as follows to underscore that consumption is a function of wealth, of the elasticity of consumption, and of the elasticity of the ratio of state wealth and organized-crime wealth:

$$c_t = \left[(1-\varepsilon-\lambda) X\right]^{-1/\varepsilon} r_t \qquad [14]$$

If equation [13] is derived with respect to the stock of weapons (q), we have:

$$\begin{aligned} &(1-\varepsilon-\lambda) X \left(r_t^*\right)^{\lambda} r_t^{-\varepsilon-\lambda} F'\left((1-q) r_t\right) \\ &+(1-\varepsilon-\lambda)(-\varepsilon-\lambda) X \left(r_t^*\right)^{\lambda} r_t^{-\varepsilon-\lambda-1} F\left[(1-q) r_t\right] F'\left[(1-q) r_t\right] \sigma_y^2 \\ &+\lambda(1-\varepsilon-\lambda) X \left(r_t^*\right)^{\lambda-1} r_t^{-\varepsilon-\lambda} r_t^* F'\left[(1-q) r_t\right] \sigma_{WV} = 0 \end{aligned}$$

If we now substitute the corresponding values of $F(\cdot)$ and $F'(\cdot) = -Ar$ in [13], it follows that:

$$\begin{aligned} &(1-\varepsilon-\lambda) X \left(r_t^*\right)^{\lambda} r_t^{-\varepsilon-\lambda} (-A r_t) \\ &+(1-\varepsilon-\lambda)(-\varepsilon-\lambda) X \left(r_t^*\right)^{\lambda} r_t^{-\varepsilon-\lambda-1} A (1-q) r_t (-A r_t) \sigma_y^2 \\ &+\lambda(1-\varepsilon-\lambda) X \left(r_t^*\right)^{\lambda-1} r_t^{-\varepsilon-\lambda} r_t^* (-A r_t) \sigma_{WV} = 0 \end{aligned}$$

This previous equation can be rewritten as:

$$\begin{aligned} &-(1-\varepsilon-\lambda) X \left(r_t^*\right)^{\lambda} r_t^{1-\varepsilon-\lambda} - \lambda(1-\varepsilon-\lambda) X \left(r_t^*\right)^{\lambda} r_t^{1-\varepsilon-\lambda} \sigma_{WV} \\ &+(1-\varepsilon-\lambda)(\varepsilon+\lambda) X \left(r_t^*\right)^{\lambda} r_t^{1-\varepsilon-\lambda} A (1-q) \sigma_y^2 = 0 \end{aligned} \qquad [15]$$

If divided by $Xr^{1-\varepsilon-\lambda}r^{*\lambda}$, we see that:

$$-(1-\varepsilon-\lambda)-\lambda(1-\varepsilon-\lambda)\sigma_{WV}+(1-\varepsilon-\lambda)(\varepsilon+\lambda)A(1-q)\sigma_y^2=0 \qquad [16]$$

From equation [16] we obtain the optimal stock of weapons and security-related technological equipment, which is:

$$q=\frac{-(1+\lambda\sigma_{VW})}{(\varepsilon+\lambda)\sigma_y^2 A}+1$$

By including equations [14] and [16] in [13], it can be deduced that:

$$\begin{aligned}
&\left[(1-\varepsilon-\lambda)X\right]^{-(1-\varepsilon)/\varepsilon}\frac{r_t^{1-\varepsilon}}{1-\varepsilon}\left(\frac{r_t}{r_t^*}\right)^{-\lambda}-\beta X r_t^{1-\varepsilon-\lambda}\left(r_t^*\right)^{\lambda}+\lambda X r_t^{1-\varepsilon-\lambda}\left(r_t^*\right)^{\lambda-1}\alpha_{r^*}r_t^*\\
&+Xr_t^{1-\varepsilon}\left(\frac{r_t}{r_t^*}\right)^{-\lambda}(1-\varepsilon-\lambda)\left[A(1-q)-((1-\varepsilon-\lambda)X)^{-1/\varepsilon}-s_t^*\right]\\
&+X(1-\varepsilon-\lambda)\lambda(1-\lambda)r_t^{1-\varepsilon}\left(\frac{r_t}{r_t^*}\right)^{-\lambda}\sigma_{VW}A \qquad [17]\\
&-\frac{1}{2}(1-\varepsilon-\lambda)(\varepsilon+\lambda)X(1-q)^2 r_t^{1-\varepsilon}\left(\frac{r_t}{r_t^*}\right)^{-\lambda}\sigma_y^2A^2\\
&+\frac{1}{2}\lambda(\lambda-1)\sigma_{r^*}^2 X r_t^{1-\varepsilon}\left(\frac{r_t}{r_t^*}\right)^{-\lambda}=0
\end{aligned}$$

From the previous equation we have:

$$\left[(1-\varepsilon-\lambda)X\right]^{-1/\varepsilon}=\frac{(1-\varepsilon-\lambda)\frac{1}{2}\left[(\varepsilon+\lambda)(1-q)^2\sigma_y^2A^2-2\lambda(1-q)\sigma_{WV}A-2A\left[(1-q)+\frac{s_t^*}{r_t}\right]\right]}{(1-\varepsilon-\lambda)\varepsilon/(1-\varepsilon)}$$
$$+\frac{\beta-\lambda\alpha_{r^*}-\frac{1}{2}\lambda(\lambda-1)\sigma_{r^*}^2}{(1-\varepsilon-\lambda)\varepsilon/(1-\varepsilon)} \qquad [18]$$

By substituting equation [18] in [14], we obtain the consumption/wealth ratio, which explains the growth of wealth and consequently the rate of GDP growth:

$$\frac{c_t}{r_t}=\frac{\beta-\lambda\alpha_{r^*}+(1-\varepsilon-\lambda)\frac{1}{2}\left[(\varepsilon+\lambda)(1-q)^2\sigma_y^2A^2-2\lambda(1-q)\sigma_{WV}A-2A\left[(1-q)+\frac{s_t^*}{r_t}\right]\right]-\frac{1}{2}\lambda(\lambda-1)\sigma_{r^*}^2}{(1-\varepsilon-\lambda)\varepsilon/(1-\varepsilon)}$$

In a fashion similar to the first model, the rate of GDP growth is denoted by ω_2, which is given by the expected value of the growth rate of wealth:

$$\omega_2=E\left(\frac{\mathrm{d}r_t}{r_t}\cdot\frac{1}{\mathrm{d}t}\right)$$

In this case the transversality condition for the consumption/wealth ratio to be positive is determined by:

$$\lim_{t\to\infty}\mathrm{E}\left[\rho r_t^{1-\varepsilon-\lambda}\left(r_t^*\right)^{-\lambda}e^{-\beta t}\right]$$

which guarantees that $c_t/r_t>0$.

Theoretical results and analysis

The first result focuses on finding the ratio of change in the rate of GDP growth with respect to the changes in the average tendency of wealth belonging to organized crime, *i.e.*:

$$\frac{\partial\omega_2}{\partial\alpha_{r^*}}=\frac{\partial\left[A(1-q)-\frac{c_t}{r_t}-\frac{s_t^*}{r_t}\right]}{\partial\alpha_{r^*}}=\frac{\lambda(1-\varepsilon)}{(1-\varepsilon-\lambda)\varepsilon}$$

Thus we have that when $\lambda<0$ and $0<\varepsilon<1$, then $\partial\omega_2/\partial\alpha_{r^*}<0$. Further, if $\lambda>0$ and $\varepsilon>1$, then $\partial\omega_2/\partial\alpha_{r^*}>0$. In the first case we can conclude that an increase in the average tendency of wealth of organized crime entails an economic slowdown if the country has a relatively high elasticity of intertemporal substitution of consumption. In the second case, un increase in the average tendency of wealth of organized crime implies economic growth if the country has a relatively low elasticity of intertemporal substitution of consumption.

The stochastic impact of the wealth of organized crime on the country's economic growth is given by:

$$\frac{\partial\omega_2}{\partial\sigma^2_{r^*}} = \frac{\partial\left[A(1-q)-\frac{c_t}{r_t}-\frac{s^*_t}{r_t}\right]}{\partial\sigma^2_{r^*}} = \frac{\lambda(\lambda-1)(1-\varepsilon)}{2(1-\varepsilon-\lambda)\varepsilon}$$

Therefore $\partial\omega_2/\partial\sigma^2_{r^*} < 0$ when $\varepsilon > 1$ and $0 < \lambda < 1$. Also, when $0 < \varepsilon < 1$ and $-1 < \lambda < 0$ or when $\varepsilon > 1$ and $\lambda > 1$, then $\partial\omega_2/\partial\sigma^2_{r^*} > 0$. Thus, if the elasticity of intertemporal substitution of consumption is relatively high, the effect of an increase in volatility of the wealth of organized crime (its stock of weapons and capital) will mean that the country will reduce its consumption and invest more both in security-related matters and in weapons, technology, human resources, and capital accumulation, leading to a positive GDP growth rate. On the other hand, with a relatively low elasticity of intertemporal substitution of consumption, the country will increase its consumption expenditure and will reduce investment both in security-related matters and weapons, technology, human resources, and capital accumulation, which will induce a drop in GDP growth as a result of an increase in the volatility of the wealth of organized crime.

In the model proposed, a value of $\lambda > 1$ can also lead to GDP growth even with a relatively low elasticity of intertemporal substitution ($\varepsilon > 1$). The stochastic impact of the country's output yield (σ^2_y) on economic growth does not appear to be straightforward, but when we assume that $\sigma_{VW} = 0$, its impact due to a change in σ^2_y affects GDP growth in the following way:

$$\frac{\partial\omega_2}{\partial\sigma^2_y} = \frac{-A^2-(1-\varepsilon)}{2\varepsilon(\varepsilon+\lambda)\left(\sigma^2_y\right)^2 A^2}$$

Note that if $0 < \varepsilon > 1$ and $-1 < \lambda < -\varepsilon$, then $\partial\omega_2/\partial\sigma^2_y > 0$. Further, if $0 < \varepsilon > 1$ and $-\varepsilon < \lambda < 0$ or $\varepsilon > 1$ and $\lambda > 0$, then $\partial\omega_2/\partial\sigma^2_y < 0$. The analysis of the stochastic impact on the marginal change in output is similar to what was mentioned previously, *i.e.*, growth is a function of the elasticity of intertemporal substitution of consumption; a relatively high elasticity of substitution leads to growth, while a low elasticity leads to economic decline.

Empirical evidence

It is not feasible to determine a specific percentage of expenditure that the government appropriates for fighting narcotics trafficking and organized crime or its volatility. Although during 2006-2011 the government spent around 174 billion pesos just on the war against narcotics trafficking and organized crime, expenditures of the latter actor are impossible to estimate; thus a model that includes both aspects, as mentioned in the theory section, is not feasible. Yet, in order to analyze some of the functional relationships that arise from the theoretical model, we propose a VAR model that uses existing data, such as security-related budgets.

In what follows we ask if an empirical relationship exists between the security-related budget and economic growth, and to this effect we use quarterly GDP figures in current prices from the INEGI data base. As a proxy for security-related expenditure we used the annual budget appropriated for government agencies tasked with national security, with data from INEGI's *Anuario Estadístico* 2012; yet to estimate the missing data, we carried out a Monte Carlo simulation in order to standardize the frequency of the series (*i.e.*, all on a quarterly basis). Given the tendency of the series in security-related expenditures, the simulation was carried out assuming that expenditures behave thusly:

$$s_t = e^{\left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma W_t} \qquad [19]$$

in other words, security expenditures have an exponential tendency and normal fluctuations. Before undertaking any specification, we summarize the basic characteristics in levels of the series that were used, where GS is the expenditure in security; these are quarterly series expressed in current prices for 2000-2012.

We can see that the behavior of the variables tends to increase from 2000 to 2012, and security expenditures have a substantially positive correlation with the increase in output.

Table 3

Characteristics of the quarterly GDP series and security expenditures

(millions of pesos at current prices)

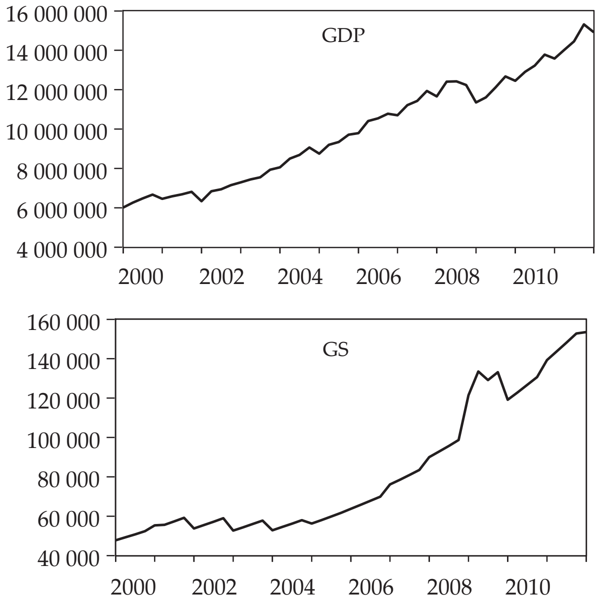
	GDP	GS
Mean	9 972 719	82 633.33
Median	9 791 537	63 759
Maximum	15 315 502	153 564.2
Minimum	6 013 121	47 752.9
Standard deviation	2 738 130	34 419.14
Skewness	0.161576	0.832558
Kurtosis	1.776502	2.13164
Jarque-Bera	3.269475	7.20026
Probability	0.195004	0.02732
Sum	4.89E+08	4 049 033
Sum of squared deviations	3.60E+14	5.69E+10
Observations	49	49

Source: compiled by the authors with EViews6 and data from INEGI, with series in levels.

Graph 2

GDP and security-related expenditures

(millions of current pesos)



Source: compiled by the authors with EViews6 and data from INEGI, with series in levels.

In Graph 2, we observe the rapid increase in security-related expenditures during the 2006-2012 period, illustrating the importance that the administration attributed to security. Several empirical analyses are available regarding public expenditure (that also include defense expenditures), such as Landau (1983), Barro (1990), and Romer (1986). The findings of these studies indicate the presence of a negative relationship between security expenditures and GDP growth. The majority of these studies are cross-sectional models, and thus it is relevant to observe what occurs with a VAR model. This model uses only two variables, the budgetary expenditure in security and GDP. Below we examine the relationship between these variables from 2000 to 2012 and ask if the relationship is causal.

VAR model

We undertake an analysis of autoregressive vectors in order to characterize the simultaneous interactions of the variables under study:

$$
\begin{aligned}
PIB &= c_1 + \sum_{p=1}^{P} \phi_p PIB_{t-p} + \sum_{k=1}^{K} \alpha_k GS_{t-k} + \varepsilon_{1t} \\
GS &= c_1 + \sum_{p=1}^{P} \phi_p PIB_{t-p} + \sum_{k=1}^{K} \alpha_k GS_{t-k} + \varepsilon_{2t}
\end{aligned}
\qquad [20]
$$

Since this technique is useful solely with stationary series, we differentiate the series to be included in this analysis so that no unit root appears, *i.e.*, with convergence both in the mean and the variance.

Unit-root test

The series used to build the VAR are shown below; various unit-root tests were carried out in order to find their degree of integration (see Table 4).

Specifications of the VAR

The correct estimation of the VAR model fulfills certain specifications, such as a zero order of integration of the series and the correct specification of the stochastic term (error) (see Table 5).

Table 4
Unit-root tests

Variable	Model	Test			
		ADF	DF-GLS	PP	KPSS
GDP	1	**2.354187**		**11.46901**	
	2	**–0.36472**	**0.352622**	**–0.752512**	**0.916895**
	3	**–2.388458**	**–2.425243**	**–2.706813**	0.123685
ΔGDP	1	**–1.233175**		–6.536803	
	2	**–2.699686**	–2.372601	–9.385508	0.373484
	3	**–2.663153**	**–2.35241**	–9.268051	**0.376098**
Δ^2GDP	1	–10.56185		–36.44477	
	2	–10.43454	**–1.233177**	–35.71674	0.196999
	3	–10.24629	**–0.118868**	–35.07948	**0.180732**
GS	1	**3.353887**		3.321489	
	2	**0.562569**	**1.620566**	**0.617157**	0.852379
	3	**–1.394945**	**–1.257592**	**–1.382605**	0.207903
ΔGS	1	–5.435977		–5.446905	
	2	–6.44147	–6.505888	–6.43071	0.247898
	3	–6.566457	–6.602195	–6.564613	0.091409
Δ^2GS	1	–7.294374		–25.11167	
	2	–7.206346	–7.266287	–24.57464	**0.1756**
	3	–7.119126	–7.183841	–24.25896	0.173723

Note: the following tests were undertaken: Augmented Dickey Fuller (ADF), Phillips-Perron (PP), Dickey-Fuller with generalized least squares (DF-GLS) and Kwatkowsky, Phillps, Schimidt and Shin (KPSS) with three different models: 1) with no intercept and no tendency, 2) with an intercept, and 3) with an intercept and a tendency. Numbers in bold type indicate that the unit root test is not significant at a 95% level of confidence.
Source: Compiled by the authors with EViews6 and data from INEGI.

Table 5
Tests of the stochastic term

Normality test				Autocorrelation test		
Component	*Jarque-Bera*	*df*	*Probability*	*Lags*	*LM statistic*	*Probability*
1	2.72461	2	0.2561	1	1.217422	0.8752
2	0.987375	2	0.6104	2	0.703347	0.9509
Joint	3.711985	4	0.4464	3	6.91795	0.1403
				4	9.742297	0.045
				5	2.91849	0.5716
				6	2.018734	0.7323
				7	1.344577	0.8538

Joint heteroscedasticity test

Chi-squared	*df*	*Probability*
52.51024	48	0.3035

Source: compiled by the authors with EViews6 and data from INEGI.

The 3.7 value of the Jarque-Bera statistic and the associated probability show that the error is normal. Regarding the heteroscedasticity test, given the associated probability, the null hypothesis of homoscedasticity is not rejected, and so the variance in the stochastic term is constant; finally, given the associated probabilities, the null hypothesis of no autocorrelation cannot be rejected, and so the stochastic term does not have this characteristic.

Stability of the model

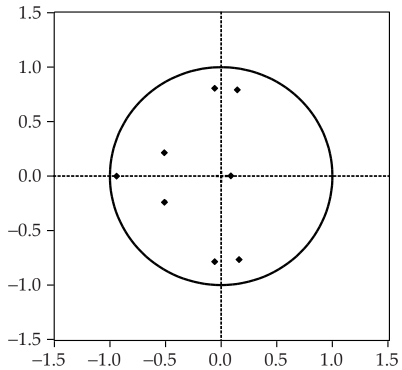
In what follows we show that the model converges and is stable and, thus, can be inverted.

Table **6**
Test of stability

Root	Module
–0.93993	0.93993
–0.049687 – 0.819676*i*	0.821181
–0.049687 + 0.819676*i*	0.821181
0.142328 – 0.802607*i*	0.815129
0.142328 + 0.802607*i*	0.815129
–0.539506 – 0.240791*i*	0.590802
–0.539506 + 0.240791*i*	0.590802
0.062355	0.062355

Note: no root is located outside the unit circle. VAR satisfies the condition of stability.
Source: compiled by the authors with EViews6 and data from INEGI.

Graph **3**
Test of stability



Source: compiled by the authors with EViews6 and data from INEGI.

Table 7 shows the variance decomposition in the VAR model.

Table 7
Variance decomposition

Variance decomposition of *D2LGDP*:				Variance decomposition of *D2LGS*:			
Period	*Standard error*	*D2LGDP*	*D2LGS*	*Period*	*Standard error*	*D2LGDP*	*D2LGS*
1	0.031279	100	0	1	0.056362	7.578877	92.42112
2	0.04113	98.23996	1.760043	2	0.074326	5.179841	94.82016
3	0.042072	94.22668	5.773325	3	0.074779	6.010747	93.98925
4	0.042686	94.0433	5.956705	4	0.074975	6.375186	93.62481
5	0.04821	94.98531	5.014687	5	0.076285	8.141895	91.8581
6	0.051223	95.5361	4.463895	6	0.078653	13.27715	86.72285
7	0.051768	94.89547	5.104529	7	0.078706	13.26083	86.73917
8	0.052486	94.99763	5.002369	8	0.078783	13.40962	86.59038
9	0.054458	95.20715	4.792853	9	0.079773	15.54677	84.45323
10	0.055651	95.40346	4.596537	10	0.080634	17.33712	82.66288
15	0.058495	95.54933	4.450673	15	0.081469	18.91736	81.08264
20	0.06006	95.74872	4.251281	20	0.081861	19.64501	80.35499

Source: compiled by the authors with EViews6 and data from INEGI.

The previous table shows the percent variation of GDP due to a percent change in security-related expenditures. There is no change in the first semester, yet from the second semester on, we can observe a positive effect on GDP due to an increase in security-related expenditures. With respect to the latter, we see it has an immediate and growing effect due to percentage variations of GDP. Based on this table we might conclude that security expenditures had a positive effect on GDP growth; nonetheless, the test for causality in the sense of Granger indicates that these are not causal variables.

Representation of the VAR

This model was prepared with the EViews6 programs and has the following structure:

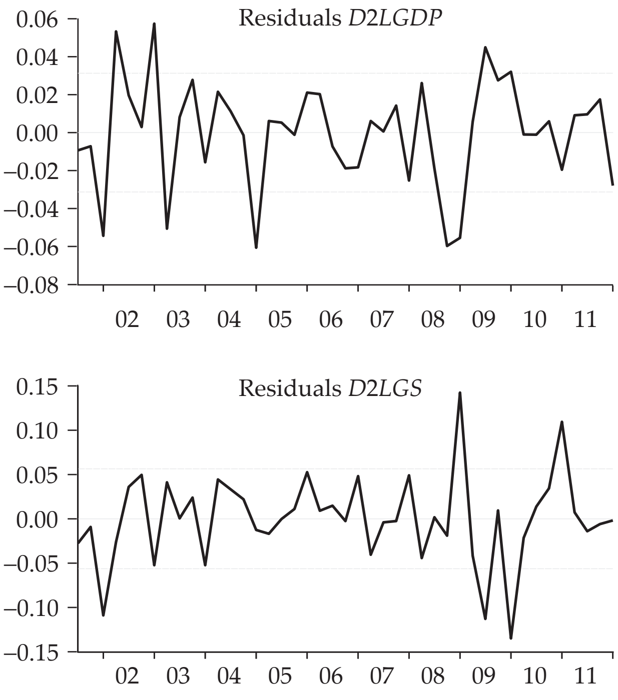
$$\begin{aligned} D2LGDP = & -0.885782911425 * D2LGDP(-1) - 0.694267130399 * D2LGDP(-2) \\ & - 0.703001056774 * D2LGDP(-3) - 0.0103945779442 * D2LGDP(-4) \\ & - 0.100702688008 * D2LGS(-1) - 0.0213212740244 * D2LGS(-2) \\ & + 0.0116368957388 * D2LGS(-3) - 0.0383009275172 * D2LGS(-4) \\ & + 0.000749899258375 \end{aligned}$$

$$
\begin{aligned}
D2LGS = {} & -0.654675639412*D2LGDP(-1) - 0.895209974462*D2LGDP(-2) \\
& - 0.787238022595*D2LGDP(-3) - 0.290465991498*D2LGDP(-4) \\
& - 0.88552243302*D2LGS(-1) - 0.772597459374*D2LGS(-2) \\
& - 0.553932920068*D2LGS(-3) - 0.18847862776*D2LGS(-4) \\
& + 0.000627967835319
\end{aligned}
$$

Although the stationary series with which the VAR was prepared show no causality in the sense of Granger, there is a strong relationship when the series are in levels, because security expenditures are explained by the GDP with up to eight lags; further, when the series are stationary, we see that a drop in GDP during 2008-2009 leads to a significant increase in security expenditures (see Graph 4).

Graph **4**

Residuals of the GDP and the GS



Source: compiled by the authors with EViews6 and data from INEGI.

Conclusions

This paper examined the effect of security-related expenditures on the growth of GDP by means of a stochastic endogenous-growth model with utility-maximizing actors. In addition, the weapons race was analyzed (including all technological resources for fighting organized crime). The analysis shows that strong (weak) growth in expenditures by organized crime in its fight against the state leads the latter to react by also increasing (decreasing) its security expenditures, which in turn entails growth (no growth) in the economy as a function of the country's elasticity of intertemporal substitution of consumption. The model also allows us to analyze the influence of volatility on the expenditure of organized crime, which may lead to economic growth when the elasticity of intertemporal substitution of consumption is relatively high, due to the government's imminent response; also, the stochastic impact on GDP performance may stimulate economic growth. In conclusion, the theoretical relationship shown to exist between security-related expenditures and GDP growth can be either negative or positive. Finally, the empirical analysis demonstrated that a drop in GDP during 2008-2009 led to a significant increase in security expenditures, which coincides with the belief that slow growth leads to higher levels of criminality and, consequently, to an increase in security expenditures. Note that although security expenditures and growth are positively correlated, and although it seems that GDP growth might be explained in part by the former, the Granger causality test leads us to conclude that no causal relationship exists between the budget expenditure in security and GDP growth.

With a security-related expenditure of 308 billion pesos during 2006-2012, we can see in Graph 1 that as this war budget grew, so too did violence. In 2011, INEGI published its *Encuesta Nacional de Victimización y Percepción sobre Seguridad Pública* (Envipe) [National Survey of Victimization and Awareness of Public Security] as part of the proceedings of the *Subsistema Nacional de Información de Gobierno, Seguridad Pública e Impartición de Justicia* (SNIGSPIJ) [National Subsystem of Government, Public Security and Law Enforcement Information]. This study concludes that from 2005 to 2011, Mexicans' awareness of insecurity had increased. In 2005, 54.2% of Mexicans were aware of the lack of security in their state of residence. In 2011, 69.5% of those interviewed felt they lived in an environment of insecurity, a fact that indicates that a conversation needs to take place regarding where the Mexican state should place its priorities.

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