NETWORK ANALYSIS OF THE MEXICAN STOCK MARKET[1]

Arturo Lorenzo-Valdés
Universidad Popular Autónoma del Estado de Puebla (Mexico)
E-mail: arturo.lorenzo@upaep.mx

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ABSTRACT

This study investigates the dynamics of equity networks in Mexico from 2018 to 2023, focusing on the impact of the COVID-19 pandemic. Methodological steps include calculating stock returns, estimating annual GARCH models, constructing lower-tailed dependency matrices, and forming networks based on these matrices. The characteristics of the resulting networks are described. In addition, 10,000 Erdos-Reyni simulations are performed to estimate GNAR models up to order two, selecting the best estimates according to AIC, BIC, and llk criteria. The predictive performance of GNAR models compared to univariate AR and VAR models is evaluated. These stages help to better understand the interconnection between Mexican financial markets, offering valuable insights for risk management and decision-making.

Keywords: Multivariate time series, networks, GARCH, GNAR.
JEL classification: G01, G32, C22, C51, C63.

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ANÁLISIS DE REDES DEL MERCADO MEXICANO ACCIONARIO

RESUMEN

Este estudio investiga la dinámica de las redes accionarias en México de 2018 a 2023, focalizándose en el impacto de la pandemia de COVID-19. Los pasos metodológicos incluyen el cálculo de rendimientos accionarios, la estimación de modelos GARCH anuales, la construcción de matrices de dependencia en colas inferiores y la formación de redes basadas en estas matrices. Se describen las características de las redes resultantes. Adicionalmente, se realizan 10,000 simulaciones Erdos-Reyni para estimar modelos GNAR hasta orden dos, seleccionando las mejores estimaciones según criterios AIC, BIC y llk. Se evalúa el desempeño predictivo de los modelos GNAR en comparación con modelos AR y VAR univariados. Estas etapas ayudan a comprender mejor la interconexión entre los mercados financieros mexicanos, ofreciendo información valiosa para la gestión de riesgos y la toma de decisiones.

Palabras clave: series de tiempo multivariadas, redes, GARCH, GNAR.
Clasificación JEL: G01, G32, C22, C51, C63.

1. INTRODUCTION

The interconnectedness and dynamics of financial markets are central economic and financial research themes. Equity networks, which represent the relationships between different securities and assets, offer a unique perspective for understanding the complexity and interdependence of financial instruments. This study focuses on equity networks in Mexico, exploring their evolution from 2018 to 2023, with a particular focus on the impact of the COVID-19 pandemic and the search for possible causes of contagion among the different components of the stock market.

In recent years, Mexican financial markets have experienced significant fluctuations influenced by domestic and international economic and geopolitical events. The COVID-19 pandemic, which began to affect globally in 2019 and continued to impact public health and economic stability in the years that followed, has been a crucial factor in the evolution of markets. Understanding how these dynamics are reflected in shareholder networks is essential for anticipating risks, improving decision-making, and strengthening the financial system's resilience.

The main objective of this study is to describe and analyze the shareholder networks in Mexico from 2018 to 2023, paying particular attention to the influence of the COVID-19 pandemic on the dynamics of these systems. It seeks to identify patterns, structural changes, and possible contagions between different sectors or companies. To achieve this purpose, various methodologies will be employed, including GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models, copula measurements, and network construction techniques, as well as estimation of global simulation-based GNAR (Generalized Autoregressive Network) models, to explain the dependence between the different actions and contrast such models with Autoregressive Vectors (VAR) and univariate time series models.

The remainder of this paper is divided as follows. The second section briefly reviews the literature on networks applied to financial markets. Subsequently, the different techniques of time series analysis are discussed. This section will detail the application of GARCH models to analyze the volatility of equity returns, allowing the identification of periods of increased risk and the assessment of the persistence of shocks in the market that will describe the marginal behavior of the stocks under study. Copulas will also be used to model the nonlinear dependencies between the returns of different actions. This will facilitate the understanding of extreme relationships and the assessment of co-mobility in times of crisis. This allows us to describe the construction of financial networks. The section will address how equity networks will be constructed, using similarity measures between stock returns. Approaches based on correlations and covariances and more advanced measures such as mutual information will be explored. Subsequently, the GNAR models used in the estimation will be described. In the fourth part, details will be provided on the data used, the temporal frequency of observations, and the selection of assets. The results of the GARCH and copula models will be presented, highlighting patterns of volatility and dependencies between returns. The construction and evolution of the shareholder networks will be analyzed in the defined time context, and the estimation of the simulation-based models. The paper's final section will summarize the main conclusions derived from the analysis of shareholder networks in Mexico during the study period.

2. LITERATURE REVIEW

Research in risk analysis and financial networks has led to several significant studies employing various techniques and focusing on different markets. Generally, studies can be grouped according to the techniques used or the markets studied. Several authors have used graphical models and network approaches to analyze the interconnectedness of markets and financial institutions. These studies have shed light on the structure of interdependence and the contagion of systemic risk.

Among the works that involve models based on graph theory and network approach, we can highlight that of Wainwright and Jordan (2007). The authors present a variational approach to address probability calculation problems in probabilistic graphical models. This work has laid the foundations for using network models to represent and estimate complex relationships in financial markets. Giudici and Spelta (2016) apply Gaussian graphical models to study interconnections in international financial flows. They also propose Bayesian graphical models to analyze networks of financial interdependence. Giudici, Sarlin and Spelta (2020) base their research on data from the Bank for International Settlements and reveal the existence of clusters of core countries in the spread of risk. Their approach identifies distinct groups of countries, providing valuable insights into the contagion of systemic risk.

Clemente, Grassi and Hitaj (2021) reveal the role of network structure in financial markets in improving the portfolio selection process. Its network-based approach has shown that optimal portfolios primarily comprise peripheral assets, effectively balancing return and risk.

How financial networks are built and interconnected is essential. Among the fundamental approaches is the construction of networks based on the correlation and partial correlation of asset returns, which have also been widely used. Tse, Liu and Lau (2010) constructed complex networks to study the correlations between closing stock prices in U.S. markets over two periods. This innovative approach has revealed the structure of interdependence in the U.S. stock market. Similarly, Kennett *et al.* (2010) propose building financial networks based on the partial correlation of the returns of stocks traded on the New York Stock Exchange. Their approach reveals the interdependencies between different market sectors. In Millington and Niranjan (2020), correlation and partial correlation networks are explored in the context of the S&P 500. Centrality measures and community detection provide valuable insights into market structure.

Shen and Zheng (2009) investigate the universal structure of interactions in financial dynamics. Their analysis of the cross-correlation matrix of price returns reveals differences in the interdependence between the Chinese, U.S., and Indian markets. On the other hand, the network construction approach based on tail dependency (used in this work) has allowed the analysis of extreme fluctuations. Wen, Yang and Zhou (2019) focus on the tail dependency structure in the forex market.

The construction of upper and lower tail dependency networks reveals the importance of considering different topological features in market situations. On the other hand, Härdle, Wang and Yu (2016) propose a semi-parametric measure to estimate the systemic interconnection between financial institutions based on tail events. Their network analysis highlights different groups' roles in the financial industry during the financial crisis. Wang and Xie (2016) analyze the tail dependency structure in the foreign exchange market using upper and lower tail dependency networks. Their approach highlights the importance of considering extreme interactions in the market.

Huynh, Foglia and Doukas (2022) study focuses on the risk of tail contagion in the Eurozone. Their research highlights connectivity and risk transmission between different sectors, providing crucial risk management and decision-making insights. In the Mexican context, Treviño (2020) aims to study and characterize the interdependence structure of the Mexican Stock Exchange (BMV) from 2000 to 2019. This study provides an overview of the interconnecting network of stocks in the BMV using correlation and concentration matrices.

Recent research on the interconnection between financial markets through network analysis has been carried out by Xu and Li (2023) and You *et al.* (2024). The first examines the relationship between participation in international trade and the connectivity of stock markets in eleven major economies, using the input-output network approach and the Diebold and Yilmaz Connectivity Index. The second explores the spillover effects of foreign capital on the Chinese stock market, highlighting a significant connection with global markets, with the US, Hong Kong, and the UK as the primary transmitters of risk.

This study also identifies other global events, such as Brexit and the US-China trade war, as sources of contagion in addition to COVID-19. In a similar vein, Wang, Wen and Gong (2024) analyze systemic contagion from the oil market to financial markets, highlighting a significant increase during oil crises.

Finally, Lorenzo-Valdes (2024) uses regular vine copulas to evaluate the dependence between the US and Latin American financial markets during COVID-19 and in subperiods (pre-COVID, COVID, post-COVID), concluding that the contagion routes have the US as the root node.

3. METHODOLOGY

The following steps are performed to assess interdependence in the Mexican stock market. First, a time series of stock returns is constructed, and an individual model is filtered for each with an equation for the conditional mean of returns and an equation for the variance of returns. As a result, a time series of standardized residuals are obtained.

Subsequently, standardized residuals measure the interdependence between the different actions. For this purpose, the dependency on lower tails is used. A measure emerged, initially, from the concept of copulas, and a matrix of this measure is constructed containing the measure between each pair of actions. Thirdly, a network is built between the actions, taking into account a threshold of the probability of tail dependence considered. A descriptive network analysis is performed for each study year from 2018 to 2023.

3.1. The Mexican stock exchange

The BMV is a financial institution that operates as a secondary market, *i.e.*, a market in which investors can buy and sell financial securities that have already been issued. Its primary function is to facilitate trading financial instruments, such as stocks, bonds, and other related products, allowing investors to buy and sell these assets against each other.

In the BMV, 143 companies are listed, of which only those that have had regular operations in recent periods were taken, leaving 101 shares. From these, daily closing prices (P) were taken and continuous returns per period (r) were calculated:

$$r_{it} = \ln P_{it} - \ln P_{it-1} \quad [1]$$

Where i is the stock listed on the BMV, and t is the time in days.

3.2. Behavior of marginal distributions

The model that describes the behavior for marginal distributions is an AR(1)-TGARCH (1,1) model.

$$\begin{aligned} r_{it} &= \varphi_{i0} + \varphi_{i1} r_{it-1} + u_{it} \\ u_{it} &= \sigma_{it} \varepsilon_{it} \\ \sigma_{it}^2 &= \alpha_{i0} + \alpha_{i1} u_{it-1}^2 + \beta_i \sigma_{it-1}^2 + \gamma_i I(u_{t-1} < 0) \end{aligned} \quad [2]$$

The perturbations ε_{it} are distributed as an asymmetric standardized Student's t, and the degrees of freedom are estimated (ν). This asymmetric probability function is constructed following the methodology of Fernandez and Steel (1998). The relevance of this methodology is that it allows the transformation of symmetric distributions into asymmetric distributions in a straightforward way, for which it is only necessary to use a scalar ξ, *i.e.*, a skewness parameter, to make this transformation. Particularly here, it transforms a Student's t-density distribution into an asymmetric Student's t-distribution.

Mathematically, this transformation is proposed considering the Student's t density function with zero mean, variance one, and degrees of freedom ν, where $\nu > 2$. The proposed methodology introduces asymmetry in the density distributions using inverse scale factors in the positive and negative values of the perturbations. These factors are defined by the scalar $\xi > 0$ (asymmetry parameter). Precisely, if this scalar is fixed, the density function for a variable x_t that is distributed following an asymmetric Student's t-density distribution is defined as:

$$f(x_t \mid \xi) = \frac{2}{(\xi + 1/\xi)} \left\{ f\left(\frac{x_t}{\xi}\right) I_{[0,\infty)}(x_t) + f(\xi x_t) I_{(-\infty,0)}(x_t) \right\} \quad [3]$$

Where

$$I_A = \begin{cases} 1 & \text{if} \quad A \\ 0 & \text{if not} \quad A \end{cases}$$

is an indicator function that takes the value of one if condition A is given. In the case of equation [3], A is the set of non-negative values of x_t in the first case and negative values of x_t in the second one.

Function [3] generalizes the Student's t-density distribution based on the asymmetry ξ (scalar) parameter. If $\xi = 1$, the resulting function is the symmetric Student's density t-distribution. If $\xi \neq 1$ the resulting function is the asymmetric Student's t-density distribution. Particularly, if $\xi < 1$, the function is a left-skewed distribution; if $\xi > 1$, the function is a right-skewed distribution. Therefore, the bias of function [3] depends on the values of ξ.

Model [2] presents an equation for returns that, in this case, is defined as an autoregressive process of order one in which the returns of the period depend on the same returns in the previous period and an equation for variance (volatility squared) that serves to describe the dispersion of continuous returns (in logarithms).

The inclusion of volatility in the models makes it possible to describe specific typical characteristics of financial time series, such as (*i*) the probability of having extreme returns higher than those that would be assumed if a normal distribution is assumed, *i.e.*, the probability distribution of returns has wider tails than a normal distribution, known as excess kurtosis; (*ii*) the leverage effect, when there is a negative correlation between performance and volatility, in the sense that when performance falls, volatility increases, and (*iii*) the time relationship of volatility that forms *clusters*, *i.e.* volatility in a period depends on volatility in previous periods. To capture these characteristics, a TGARCH (Threshold Generalized Autoregressive Conditional Heteroskedasticity model) introduced by Zakoian (1994) and Glosten, Jagannathan and Runkle (1993) is used to estimate the volatility equation as an extension to the ARCH models, initially developed by Engle (1982) and generalized by Bollerslev (1986).

3.3. Copulas and dependency measures

The dependency structure between variables can be modeled by a mechanism that expresses the cumulative distribution function (CDF) from the marginal distribution functions (MDF). A copula $C(u_1,u_2,\ldots,u_n)$ is an FDA for n uniform variables over the unit interval. Sklar's theorem (1959) says that if we take $u_j = F_j(x_j)$ for $j = 1,\ldots,n$ as the FDA of a univariate continuous random variable X_j, then $C(F_1(x_1),F_2(x_2),\ldots,F_n(x_n))$ is a multivariate distribution function for $X(X_1,X_2,\ldots,X_n)$ with marginal distributions F_j, $j = 1,\ldots,n$. Conversely, if F is a continuous multivariate FDA with univariate marginals, F_j, $j = 1,\ldots,n$, then there exists a single multivariate copula C such that $F(x_1,x_2,\ldots,x_n) = C(F_1(x_1),F_2(x_2),\ldots,F_n(x_n))$.

This allows the study of the dependence of random variables based on their marginal distributions. The properties of copulations have been studied by several authors, most notably the work of Nelsen (2006). First, they are invariant to strictly positive transformations of random variables. The second property is the consistency between the calculation of the concordance measures and the parameters of the copulas. Finally, the third property consists of the treatment that can be given to asymptotic tail dependence.

Considering the bivariate case, we want to measure the dependence between two financial assets. This can be done by calculating the asymptotic dependency on the distribution's tails. That is the measurement of the behavior of random variables during extreme events. Thus, measures emerge that indicate the probability of an extreme increase (decrease) in the returns of one financial asset, given that there is an extreme increase (decrease) in another financial asset. By the above, the dependency coefficients in lower λ^l and upper λ^S tails are defined as:

$$\begin{aligned} \lambda^l &= \lim_{\alpha\to 0^+} P\left(X_2 \langle F_2^{-1}(\alpha) \middle| X_1 < F_1^{-1}(\alpha)\right) = \lim_{\alpha\to 0^+} \frac{C(\alpha,\alpha)}{\alpha} \\ \lambda^S &= \lim_{\alpha\to 1^-} P\left(X_2 > F_2^{-1}(\alpha) \middle| X_1 > F_1^{-1}(\alpha)\right) = \lim_{\alpha\to 1^-} \frac{1-2\alpha+C(\alpha,\alpha)}{1-\alpha} \end{aligned} \quad [4]$$

There is tail independence if the values in [4] are zero, dependency if the values are between zero and one, and perfect dependence if they are equal to one. Schmidt and Stadtmüller (2006) propose a set of non-parametric estimators for the upper and lower tail coefficients. This paper uses these estimators to find a matrix of dependency coefficients in lower tails for stock returns.

3.4. Graphs and networks

Network building is used to model and analyze various financial and economic phenomena, allowing for a better understanding of the relationships and connections between different financial and economic variables. A graph is a data structure composed of nodes (points) and edges (connections) representing and visualizing the relationships between different financial elements. Nodes can represent entities such as companies, financial assets, investors, markets, or economic variables, while edges denote their connections or relationships.

In this sense, nodes can represent financial assets, and edges can indicate correlation, volatility, or other risk metrics between them. They can help us understand how companies are connected and how events in one part of the market can affect others. Therefore, it is essential for understanding the complex interactions in financial markets and making informed decisions. It enables financial professionals to identify opportunities, mitigate risks, and optimize investment and asset management strategies.

In financial networks, there are several essential measures that analysts and financial professionals use to assess the structure and dynamics of the network. These measures provide valuable insights into the interconnectedness of financial elements and can be critical for risk management, investment decision-making, and understanding the financial system's stability.

In the case of networks composed of financial actions, some of the most relevant measures would be:

- The measure of density in the context of networks refers to the proportion of connections in a network relative to the total number of possible connections. In other words, the density of a network quantifies how interconnected nodes are compared to the total number of connections that could exist. Density is expressed as a value between 0 and 1, where 0 indicates no connections, and 1 indicates all possible connections.
- The degree of a node in a financial action network represents the number of connections or links to other actions in the set. In this context, the grade of a stock indicates how many other stocks are related to it through correlations, reversals, or market movements.

- The measure of modularity in financial networks is used to assess the modular structure of a financial system, *i.e.*, how nodes (typically financial institutions) are divided and grouped into more densely connected communities or modules than the rest of the system. Modularity is a measure that quantifies the quality of this partition of the network into communities. A high modularity indicates the network is well divided into distinct communities, while a low modularity suggests a more homogeneous network. The measure of modularity in financial networks provides insight into the structure of the network and how financial institutions are organized into communities, which can be crucial for understanding the stability and resilience of the financial system as a whole.
- Betweenness Centrality: In a network of financial stocks, this measure can indicate which stocks act as critical intermediaries in the relationships and flows of information among other stocks.

These measures are critical to understanding the structure and behavior of financial equity networks, which can help investors, analysts, and portfolio managers make informed decisions and better manage risk in a stock market context.

3.5. Global Generalized Autoregressive Network Model-α

Additionally, a simulation analysis is performed to find the network by year that best fits a global Generalized Autoregressive Network Model (global NAR) proposed and developed by Knight *et al.* (2020). Consider a node time-series vector $\mathbf{X}_t = (X_{1t},\ldots,X_{Nt})'$, where N is fixed. For each node $i \in \{1,\ldots,N\}$ and time $t \in \{1,\ldots,T\}$, the global generalized autoregressive model of order $(p,[s]) \in N \times N_0^p$ for $\mathbf{X}_t$ is:

$$X_{it} = \sum_{j=1}^{p} \left(\alpha_j X_{it-j} + \sum_{r=1}^{s_j} \sum_{q \in N_t^{(r)}(i)} \beta_{jr} \omega_{iq}^{(t)} X_{qt-j} \right) + u_{it}$$

Where $p \in N$ is the maximum lag, $[s] = (s_1,\ldots,s_p)$ and $s_j \in N_0$ is the maximum neighbor dependency stage for the lag j, with $N_0 = N \cup \{0\}$, $N_t^{(r)}(i)$ is the neighboring set of the node i at stage r at the moment t, $\omega_{iq}^{(t)} \in [0,1]$

is the weight of the connection between the node i and the node q at the moment t.

The $\alpha_j \in R$ are standard autoregressive parameters for the lag j of each node. The $\beta_{jr} \in R$ corresponds to the effect of the neighbors on the r-th state, on the j lag. Perturbations $\{u_{it}\}$are assumed to be independent and identically distributed at each node i, with zero mean and variance σ_i^2.

The network may change over time in the GNAR model, but the covariates remain fixed. This means that the underlying network can be modified over time, for example, to allow nodes to enter and exit the series, but model tuning can still be done. The $\alpha_j \in R$ defines a process with the same behavior on all nodes, with differences being present only due to the graph's structure. The interpretation of the network regression parameters, β_{jr}, is the same throughout the network.

4. DATA AND RESULTS

4.1. Data

The data in the study consists of daily closing prices of stocks listed on the BMV from January 2, 2018, to October 30, 2023. At the time of the study, 143 companies were listed, of which those with positive price movements and operating volume were considered, leaving 101 companies. Of these 101 companies, the BMV classifies them into the following sectors: industrial (26 companies), consumer products (17 companies), financial services (17 companies), materials (16 companies), consumer goods services (14 companies), telecommunications services (6 companies), health (3 companies) and energy (2 companies).

The sample is divided into the six years considered from 2018 to 2023. Daily continuous returns are calculated as in [1].

4.2. Network construction

We started by calculating the continuous returns according to [1] to build the network. For each year from 2018 to 2023 and for each of the 101 actions, an AR(1)-TGARCH(1,1) model was estimated as in [2], and standardized residuals were obtained. The latter was used to construct a matrix measuring bottom-tailed dependency using the nonparametric estimators proposed by Schmidt and Stadtmüller (2006). The distance transformation function (d) can be expressed as:

$$d_{ij} = 1 - \lambda_{ij}^{I} \quad [5]$$

For each year, these are used to compute the matrices containing the distance transformation function as in [5] based on the lower-tail dependency coefficients. This matrix is used to construct the graph considering a threshold; in Figure 1, the graphs constructed from a threshold of probability of tail dependence greater than 0.25 are presented.

Figure 1 shows the graphs of the networks obtained from 2019 to 2023. It can be seen that there is a more significant number of interconnections in the network than in 2020 when the COVID-19 pandemic was at its peak. In this network, the action nodes are shown in different sizes depending on the degree of the action. If we consider the centrality of intermediation, we would have Figure 2.

Table 1 shows the measures of the shared network by year. The network measurements in 2018 are significant compared to other years. The above is because in 2018 the Mexican stock market experienced several events and factors that impacted its performance. One of the most notable events was holding the presidential elections in July 2018. The political

Figure 1. Stock networks in the BMV. The grade determines the size

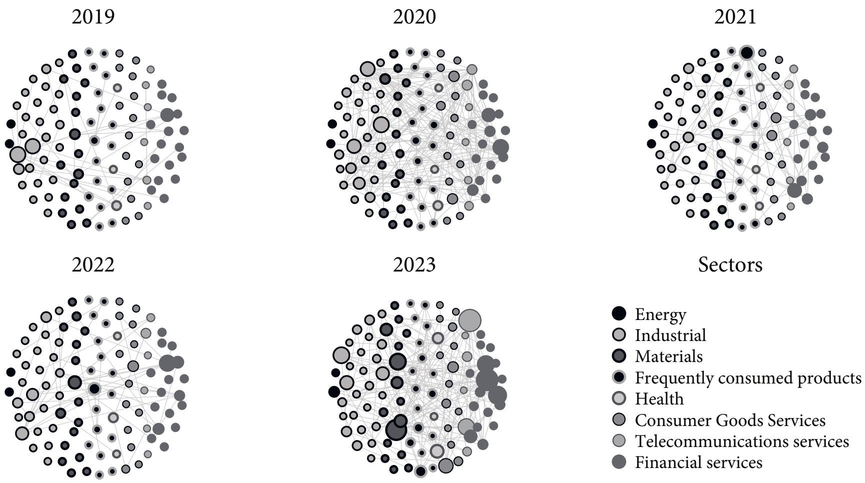
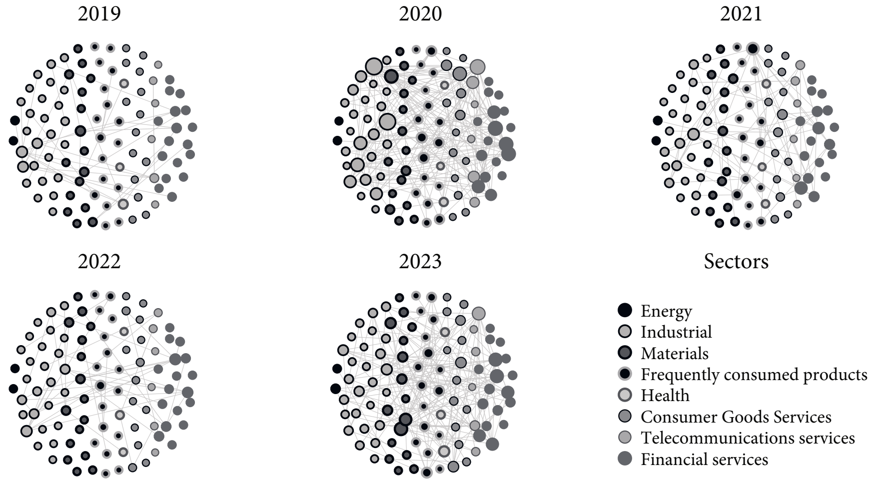


Figure 2. Stock networks in the BMV. The betweenness centrality coefficient determines the size



uncertainty of the elections and the possible direction of economic and trade policies under a new government influenced investor perceptions.

In addition to the elections, other global factors also affected financial markets, including the trade war between the United States and China, the normalization of monetary policy in the United States, and the volatility in commodity prices. Density appears to have increased significantly in 2020 (the year of the COVID-19 pandemic) and 2023 compared to previous years. This indicates that the grid became more interconnected in those years.

Table 1. Network measurements for the years 2018 to 2023

Year	Size	Density	Degree	Betweenness	Modularity
2018	101	0.0224	2.2376	23.2772	–0.0246
2019	101	0.0081	0.8119	13.1584	0.1098
2020	101	0.0386	3.8614	29.3663	0.0273
2021	101	0.0139	1.3861	14.5644	0.0607
2022	101	0.0095	0.9505	20.3564	0.0243
2023	101	0.0319	3.1881	70.6733	0.0419

The average degree also shows an increase in 2020 and 2023, suggesting that stocks on the network had more connections in those years. Betweenness shows much variation, with a significant peak in 2023. This could indicate that some specific nodes played a crucial role as bridges on the shortest path between other nodes in that year.

Modularity varies over the years but does not show a clear trend. It is important to note that a negative modularity value (as in 2018) has no inherent meaning, as modularity can vary between −1 and 1. However, values closer to 1 indicate a more modular network. Table 2 shows the companies with the highest degree and centrality of intermediation. This means that, in each of the years, these companies are the ones that act most in the process of information flow between stocks. In a financial stock network, the grade distribution shows how the degrees of the stocks are distributed. It is expected to find a small number of "lead actions" with many links and more "follower shares" with more limited connections.

The degree is highest in the nodes (stocks) of the industrial sector, followed by telecommunications and financial companies. A higher degree in the industrial sector suggests that stocks within this sector are more interconnected than in other areas, such as telecommunications and financials.

It could indicate a greater dependency or correlation and lead to contagion in the stock system when passing through the industrial sector, which could be due to economic, commercial, or industry factors

Table 2. Companies with the highest degree and betweenness for each year

Year	Firm	Sector	Degree	Betweenness
2018	GFNORTE	Financial services	15	268.87
2019	ALFA	Industrial	8	285.00
2020	ORBIA	Industrial	22	245.30
2020	VOLAR	Industrial	21	285.73
2021	HERDEZ	Frequently consumed products	10	236.83
2022	ASUR	Industrial	8	188.00
2022	GFINBUR	Financial services	6	272.50
2023	TLEVISA	Telecommunications services	13	549.48

in particular and in the case of 2020 to the pandemic and that, in this case, these stocks are more sensitive compared to stocks in the telecommunications and finance sectors.

During 2020, the COVID-19 pandemic had a significant impact on financial markets around the world, including Mexico. Although the situation changed over time, some sectors were more affected than others. Some of the hardest-hit sectors included tourism, which, with travel restrictions and health concerns, led to a significant decline in demand for air travel and tourism-related services.

The drop in oil demand and the price war between Saudi Arabia and Russia negatively affected energy companies, especially those focused on oil production. Businesses related to non-essential goods and services experienced a decline in demand as consumers prioritized essential spending and cut back on luxury or non-essential purchases. Although the broader financial sector was affected, banks faced challenges due to concerns about rising bad loans and declining economic activity.

On the other hand, some sectors showed resilience or even growth during the pandemic. Technology companies, especially those related to online services, communications, and digital solutions, saw a surge in demand as more people turned to technology to work, study, and entertain themselves from home. Health-related companies and pharmaceutical manufacturing played a vital role in responding to the pandemic, with some experiencing increased demand for health-related products and services.

Stock market contagion in 2020 was widespread and affected global markets, including Mexico. Volatility was high, with sharp declines in stock indices followed by some recovery as economic stimulus measures were implemented and progress was made in developing COVID-19 vaccines. The response of governments and central banks and progress in containing the pandemic influenced the evolution of financial markets.

By 2021, some sectors affected by the pandemic in 2020, such as tourism and airlines, were expected to recover as restrictions eased and vaccination progressed gradually. The technology and health sectors would likely continue to be resilient as reliance on technology persisted, and the focus shifted to innovation and public health response. The recovery in oil prices could have positively impacted companies in the energy sector. However, the specific direction could depend on global supply and demand.

Government policies, including economic stimulus and macroeconomic developments, could have significantly impacted financial markets, including the stock market.

Uncertainty around the evolution of the pandemic, virus variants, and other geopolitical events could have contributed to volatility in financial markets. According to the 2023 network, relations have increased due to possible effects of the Russia-Ukraine war.

4.3. Estimation of the global generalized autoregressive network model-α

We simulated 10,000 random non-directed Erdos-Renyi networks, each with a connection probability of 0.03, for each year and estimated global-α GNAR models up to $p = 2$, *i.e.*, eight models for each year, totaling 80,000 simulations per year, and chose the best models according to the AIC, BIC and llk criteria. Table 3 presents the firms with the highest degree and centrality of intermediation according to the AIC.

Table 3. Companies with the highest degree and betweenness for each year

Year	Firm	Sector	Degree	Betweenness
2018	AMX	Telecommunications services	9	631.2052
2019	GFINBUR	Financial services	8	462.0951
2019	LAMOSA	Materials	8	678.9278
2020	MFRISCO	Materials	8	645.8841
2020	LACOMER	Frequently consumed products	5	738.4067
2021	AXTEL	Telecommunications services	7	802.36
2021	GMEXICO	Materials	7	435.36
2021	PASA	Industrial	7	549.71
2022	BOLSA	Financial services	6	438.91
2022	FRAGUA	Health	6	441.31
2022	GMXT	Industrial	6	309.99
2022	INVEX	Financial services	6	372.99
2022	LAMOSA	Materials	6	541.46
2022	NEMAK	Consumer Goods Services	5	607.77
2023	GIGANTE	Frequently consumed products	8	603.42

In this part, the results of the simulation and estimation of the optimal model for each year are presented according to the AIC, BIC, and llk criteria. The centrality of degree and betweenness varies between different companies and years. In 2019, LAMOSA had a high centrality in grade and betweenness, and AXTE in 2021 had the highest intermediation, suggesting a crucial role of these two companies as a bridge in the respective networks. In 2022, several companies share a high degree, and NEMAK stands out regarding intermediation. In 2023, GIGANTE has the highest degree and the highest intermediation. These results provide insight into

Table 4. Estimation of parameters of the GNAR-global models obtained from the simulation with the best values of the AIC, BIC and llk criteria, for each year

	2018		2019		2020	
	AIC-BIC	llk	AIC-BIC	llk	AIC-BIC-llk	
α_1	−0.0414*	−0.0437*	−0.0013	−0.0010	−0.0038	
	(−0.041)	(−0.044)	(−0.001)	(−0.001)	(−0.004)	
β_{11}	0.0410*	0.0290*	0.0441*	0.0449*	0.0345*	
	(0.041)	(0.029)	(0.044)	(0.045)	(0.035)	
β_{12}	-	0.0504*	0.0536*	0.0544*	-	
	-	(0.050)	(0.054)	(0.054)	-	
α_2	−0.0078	−0.0119***	−0.0105***	−0.0087	0.0064	
	(−0.008)	(−0.012)	(−0.011)	(−0.009)	(0.006)	
β_{21}	0.0544*	0.0028	-	0.0006	0.0378*	
	(0.054)	(0.003)	-	(0.001)	(0.038)	
β_{22}	-	0.1148*	-	−0.0369**	-	
	-	(0.115)	-	(−0.037)	-	
AIC	−41.96113	−41.95792	−35.44612	−35.44185	−46.17257	
BIC	−41.90589	−41.87507	−35.39088	−35.359	−46.11749	
llk	−5,478.383	−5,476.794	−6,315.561	−6,314.109	−4,956.439	

Note: The parameters are significant at: 1% *, 5% ** and 10% ***.

the relative importance of firms in the network over time. The centrality of degree and betweenness can indicate a company's strategic position in the network, either because of its direct connectivity or because of its role as an intermediary in communication between other companies.

Table 5 presents the optimal estimates according to the GNAR models estimated in the simulations, which obtained the best score according to the Akaike, Bayesian, and maximum likelihood criteria.

The parameters between the different models are similar, indicating that the relationships between stocks and their returns are stable over

	2021			2022		2023	
	AIC	BIC	llk	AIC-llk	BIC	AIC-llk	BIC
	0.0002	–0.0002	–0.0005	–0.0224*	–0.0246*	–0.0278*	–0.0294*
	(0.000)	(0.000)	(0.000)	(–0.022)	(–0.025)	(–0.028)	(–0.029)
	0.0469*	0.0476*	0.0456*	0.0160***	0.0507*	0.0052	0.0370*
	(0.047)	(0.048)	(0.046)	(0.016)	(0.051)	(0.005)	(0.037)
	-	-	0.0211	-	-	–0.0006	-
	-	-	(0.021)	-	-	(–0.001)	-
	–0.0135**	–0.0131**	–0.0136**	–0.0167 *	–0.0188 *	–0.0010	–0.0005
	(–0.014)	(–0.013)	(–0.014)	(–0.017)	(–0.019)	(–0.001)	(0.000)
	0.0212**	-	0.0207**	–0.0351 *	-	–0.0076	0.0470*
	(0.021)	-	(0.021)	(–0.035)	-	(–0.008)	(0.047)
	-	-	-	-	-	0.0863*	-
	-	-	-	-	-	(0.086)	-
	–31.77419	–31.76793	–31.77157	–36.09249	–36.08509	–55.9008	–55.86695
	–31.7191	–31.72661	–31.70271	–36.03741	–36.04378	–55.79057	–55.79347
	–6,813.83	–6,815.638	–6,813.168	–6,256.769	–6,258.723	–2,449.969	–2,454.863

Table 5. Performance evaluation of the models RMSE and MASE

		Step- ahead			
		N = 1	*N* = 2	*N* = 5	*N* = 10
GNAR	RMSE	4.14522	4.14817	4.14498	4.14612
AIC-llk	MASE	0.90011	0.91108	0.90157	0.90068
GNAR	RMSE	4.14374	4.14721	4.14510	4.14626
BIC	MASE	0.89823	0.90533	0.90046	0.90023
ARIMA	RMSE	4.14775	4.14782	4.14621	4.14658
	MASE	0.90279	0.91352	0.90472	0.90162
VAR	RMSE	13.26796	30.68082	810.71630	82,171.20000
	MASE	10.03090	21.20091	349.33500	25,759.06000

time. However, when we plot those relationships or interconnections with the probability measure in the lower tail, the probability that there is an extreme fall in one stock, given that there is an extreme fall in another stock, increases in crisis periods, which increases contagion in those periods whether they are crises, caused by economic, health, or financial institutions.

To compare the models, the 2023 model year was used, taking as an in-sample the period from January 3 to August 31 and an out-of-sample from September 1 to November 19. The measurements to be used are the mean square and absolute errors. Generally, the model with the slightest error in the evaluation measures, such as RMSE (Root Mean Squared Error) and MASE (Mean Absolute Squared Error), is sought. Table 5 presents the performance evaluation measures of the models.

In this case, the AIC and llk-based GNAR models, BIC and Order 2 Autoregressive, have similar RMSE and MASE, but the VAR-based model has significantly higher errors. Looking at the results, the GNAR models appear to have similar errors, but the BIC-based GNAR model has slightly lower RMSE and MASE values in all forecast steps ($N = 1$, $N = 2$, $N = 5$, $N = 10$). We can say that GNAR models have better prediction performance and are an excellent tool for modeling multivariate time series and the interdependencies between financial assets.

5. CONCLUSIONS

The study was based on daily closing price data from 101 companies listed on the BMV from 2018 to 2023. These companies were selected for having positive price movements and trading volume. The sample is divided into the six years considered from 2018 to 2023. Annual networks were constructed using continuous daily yields and AR(1)-TGARCH(1,1) models, and then the dependence on lower tails on standardized yields was empirically estimated. There was a significant increase in network interactions during 2020, coinciding with the COVID-19 pandemic.

Measurements on networks show that density increased in 2020 and 2023, indicating greater interconnectedness between stocks. The average degree and betweenness also saw notable increases in 2020 and 2023, suggesting increased connectivity and the presence of crucial nodes in these years. The grade distribution showed that stocks in the industrial sector had the highest grade, followed by telecommunications and financial companies.

The COVID-19 pandemic in 2020 had a widespread impact on markets, affecting different sectors in varying ways. Technology, healthcare, and pharmaceutical companies responded positively, while sectors such as tourism, energy, and non-essential goods were affected hardest.

Networks were calculated differently, considering simulations to compare network models, and the best ones were chosen according to Akaike, Bayesian, and maximum likelihood criteria.

GNAR models are introduced to estimate what remains of interdependence between stock series. The centrality of degree and betweenness varied between companies and years, highlighting the importance of specific companies as bridges in the network. GNAR models showed good performance in grid prediction, especially the BIC-based model Measures such as mean square and absolute error were compared, concluding that GNAR models effectively model multivariate time series and interdependencies between financial assets.

In summary, the study offers a detailed view of the dynamics of the stock network on the Mexican Stock Exchange over the years, identifying significant patterns and highlighting the influence of external events, such as the pandemic and geopolitical conflicts, on the interconnectedness and behavior of stocks. In addition, GNAR models are presented as efficient tools to understand and predict these complex interrelationships in the financial market. ◀

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